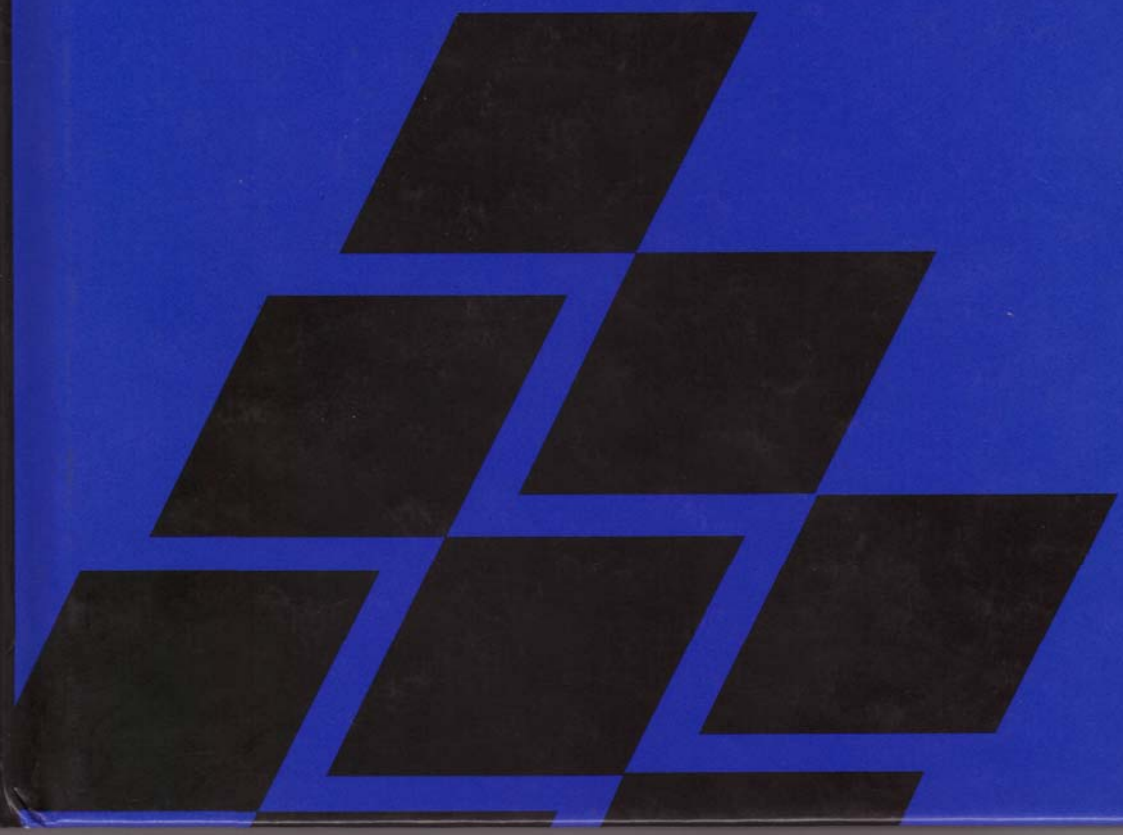
Deterministic Observation Theory and Applications

Jean-Paul Gauthier • Ivan Kupka



DETERMINISTIC OBSERVATION THEORY AND APPLICATIONS

JEAN-PAUL GAUTHIER

IVAN KUPKA

CAMBRIDGE
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Preface

A long time ago, while working on paper [19], we felt that there was a need to write a book on the subject of observability. Now, after many vicissitudes, this is a done thing.

During the conception of the book, the very novel point of view we had developed in our papers did not change. We discovered that it was really the right one, and was extremely efficient. In fact, based on it, we could build a totally new, complete, and general theory, and a new methodology for the problems related to observability, such as "output stabilization." At the same time, we applied our methodology to practical problems, and we realized that our methods were extremely efficient in practice.

At the very beginning, we intended to write a "survey" on the problems of observability, including nonlinear filtering. As the work progressed, we changed our minds. First, from the practical point of view, we faced a daunting task: a book of that type, had it ever seen the light of a day, would have been a monster. But, more important, our theory would have been drowned in a mass of disparate, disconnected facts.

Hence, this book presents only the general theory we have discovered, with a selection of real-life applications to convince the reader of the practical capability of the method. We strictly avoided the type of academic examples which are rife in many control theory publications.

Several principles guided us in the elaboration of this book:

- First, the book should be short. Including some developments in the stochastic context was a definite possibility, but this would have required the use of deep mathematical tools for meager returns. Enough mathematical theories already are used in the book.
- Second, the book is an excellent opportunity to convince people with a mathematical bent that "observation theory" is not out of place in mathematics. For that reason, the style of this book is a mathematical one. Also, we want to show that applied problems in the real world

can be dealt with by using beautiful mathematics. On the other hand, mathematics is not the main object of this book, but an excellent tool to achieve our goals.

- Third, we want to convince applied people (e.g., control engineers, chemical engineers) that our methodology is efficient. Therefore, they should strive to understand it and, above all, to use it. For this purpose, we want to point out the following:

  - We strove to make all the necessary mathematical tools accessible to uninitiated readers.
  - Bypassing the details of the proofs does not impair the understanding of the statements of the theoretical results and the constructive parts of the theory (many of the proofs are not obvious).
  - Chapter 8, containing the applications, is friendlier to the nonmathematical reader, albeit rigorous.

The development of the practical applications in this book, and of others not mentioned in it, was possible thanks to the cooperation with the French branch of the Shell company, its research center at "Grand-Couronne." One of the applications was actually implemented at the refinery of Petit-Couronne (France). The first author particularly wants to express his deep gratitude to the whole process control group there, more especially to Denis Bossane, François Deza, Marjoleine Van Doothing, and Frederic Viel, for their help, support, and for the good time we spent together. A very special and friendly remembrance goes to Daniel Rakotopara, the head of the group at that time, who so unfortunately died recently.

J-P. Gauthier expresses his warmest thanks to Jean-Jacques Dell'amico (head of the research center) and Pierre Sommelet (chief of the group), who not only took care of financial needs, but also are great friends.

Chapters 3, 4, 5, and 7 of this book contain, among others, the results of the papers [18], [19], [32]. For their kind permission to reproduce parts of papers [18], [19], and [32], we thank, respectively, the Society for Industrial and Applied Mathematics (Observability and observers for nonlinear systems, *SIAM Journal on Control*, Vol. 32, No. 4, pp. 975–994, 1994), Springer-Verlag (Observability for systems with more outputs than inputs, *Mathematische Zeitschrift* 223, pp. 47–78, 1996), and Kluwer Academic Publishers (with P. Jouan, Finite singularities of nonlinear systems. Output stabilization, observability, and observers. *Journal of Dynamical and Control Systems* 2(2), pp. 255–288, 1996).

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