

Supply chain coordination; A Game Theory approach

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Abstract

In a supply chain organized as a network of autonomous enterprises, the main objective of each partner is to optimize his production and supply policy with respect to his own economic criterion. Conflicts of interests and the distributed nature of the decision structure may induce a global loss of efficiency. Contracts can then be used to improve global performance and decrease risks. The purpose of the paper is to evaluate the efficiency of different types of contracts between the industrial partners of a supply chain. Such an evaluation is made on the basis of the relationship between a producer facing a random demand and a supplier with a random lead time. The model combines queuing theory for evaluation aspects and game theory for decisional purposes.

1 Introduction

The supply chains considered cover different enterprises sharing common information and logistic networks. Due to the distributed nature of the system and the decisional autonomy of heterogeneous decision centers, organization of tasks and activities raises some specific problems of coordination and integration.

Enterprises can be seen as players in a game defined by a common goal, but separate constraints and conflicting objectives. Taking into consideration that the entities of a supply chain need to cooperate in order to achieve the global goal, a problem appears: how to cooperate without knowing the internal models of the other entities involved? In order to obtain acceptable trade conditions, a certain form of negotiation turns out to be necessary. Game theory provides a mathematical background for modeling the system and generating solutions in competitive or conflicting situations. The basic rationality principle of game theory states that each player acts to optimally accomplish his/her individual goal, taking into account that the others play in the same manner. However, if the individual goal of each player is uniquely to maximize his gain or to minimize his loss, the agreements obtained by negotiation may be fragile and will not generally guarantee global optimality for the whole supply chain, particularly when external demand is stochastic. For these reasons, much effort has been recently devoted to conceiving contracts strengthening the commitments of partners through risk, profit or cost sharing, and/or moving the equilibrium state of the game toward a better global performance. Game theoretical applications in supply chain management are reviewed by Cachon and Netessine (2004) and Leng and Parlar (2005). Cachon (2003) reviews the literature on supply chain coordination with contracts. Examples of contract parameters that can be used to achieve coordination, are quantity discounts, returns (buy backs), quantity flexibility, and the use of subsidies/penalties. Gupta and Weerawat [7]. consider the interactions between an end-product manufacturer and an intermediate-product supplier, assuming that the manufacturer has no inventory for end-products. They describe revenue sharing contracts able to coordinate this supply chain.. Gupta *et. al.* [8] generalize this study to a system in which the manufacturer has an inventory for end-products. They propose revenue and cost sharing contracts with a delay penalty for the supplier in case of a late delivery by the manufacturer. They also construct a search technique in a closed interval to compute the optimal base stock value for the manufacturer.

This paper presents several types of contracts between a producer and a supplier: price-only contracts, backorder costs sharing through transfer payments, capacity reservation to assure supply. Each contract is characterized by several parameters, whose values are to be determined through optimization and negotiation, using a simple analytical model described in the following section.. Due to the use of such a general but somewhat simplistic model, application of the results to a real system

would require complementary studies by simulation, tests and measurements. However, it is believed that the decisional levers studied in the paper are relevant in most applied cases. It is mainly the optimal values of decision variables which remain to be calculated in practical applications.

2 Model and analysis

The basic supply chain element considered in the paper consists of one producer and one supplier. Such an element is considered generic in terms of trade agreements and product flows within a supply chain. The producer manufactures and delivers goods to the customers using raw products delivered by his supplier.

2.1 Basic assumptions

Manufacturing and delivery times of the producer are supposed negligible relatively to delivery time from the supplier. Classically, the bill of materials applies to determine the quantity of components necessary per unit of final product. But for simplicity of notations, a one-to-one correspondence in technical coefficients is assumed between the considered component and the final product. Both demand and delivery processes are supposed random with respective average rates λ and μ . Parameter λ is exogenous. Demand is supposed stationary with an average rate, λ , supposed known by the producer. Parameter μ is the main operational decision variable for the supplier. It measures the production capacity devoted to this producer. It is also one of the basic negotiation parameters between the supplier and the producer.

An additional decision variable for the producer is the reference inventory level of finite products, S . The producer is supposed to use an order-driven base stock policy. When an order comes to her, it is immediately satisfied if its amount is available in the stock. If not, it has to wait until the inventory has been sufficiently replenished by the arrival of products from the supplier. In both cases, an order is placed from the producer to the supplier whenever a demand comes and has the same amount (1 in the

unitary case). Such a base stock control policy can also be interpreted as a Kanban mechanism [2]. After an initial inventory replenishment stage, the inventory position is held constant with value S . At time t , the two random variables; the current inventory level $I(t)$ and the number of uncompleted orders $u(t)$ are linked by the relation:

$$I(t) + u(t) = S \quad (1)$$

It can be noted that under the considered inventory policy, $u(t)$ also represents the current queue length of orders for the supplier.

Consider the following notations:

I is the random variable representing the producer inventory level in stationary conditions,

u is the random variable representing the number of uncompleted orders from the producer not yet delivered by the supplier in stationary conditions,

h is the unit holding cost,

b is the unit backorder cost,

p is the retail price,

c_s is the supplier production cost per unit,

c_p is the producer production cost per unit.

Let $T(\lambda, \mu, S)$ be the expected transfer payment rate from the producer to the supplier. That function may depend on a number of quantities, according to the contract considered.

2.2 The M/M/1 model

Consider now the model of a unitary demand occurring according to a Poisson process with rate λ . The supplier delay of delivery is modeled as an exponentially distributed service time with mean value $1/\mu$, satisfying the stability condition $\rho = \lambda / \mu < 1$. Under the (S-1,S) base stock policy, the inventory position is a constant with value S and the number of uncompleted orders, $u(t)$, represents the

queue length of orders for the supplier [1]. It is a simple M/M/1 system with birth-death coefficients (λ, μ) . Accordingly, due to equation (1), the inventory level $I(t)$ is a Markov chain described by the transition graph of Fig.1.

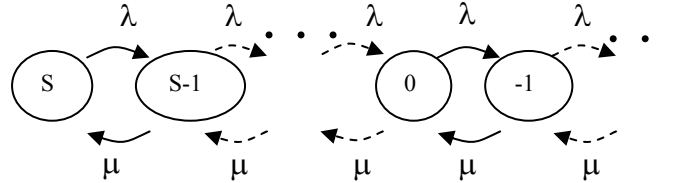


Fig. 1. Transition diagram of the Markov chain of inventory level

In stationary conditions, $P_w = \text{Prob}(u = w) = \text{Prob}(I = S - w) = \rho^w (1 - \rho)$.

The expected value of u is defined by (2):

$$Z = E[u] = \sum_{w=0}^{\infty} w P_w . \quad (2)$$

It corresponds to the expected number of customers in the M/M/1 queuing system, $Z = \frac{\rho}{1 - \rho}$. Then,

from Little's law, the expected delivery time for the producer is:

$$\tau = \frac{1}{\mu - \lambda} \quad (3)$$

It is assumed that demands arriving when the stock is empty are backordered, and a backorder cost, b , is associated to each unit rate of backordered sales. Let F be the discrete distribution function of u , and $\bar{F}(u) = 1 - F(u)$. The probability of backorder is:

$$\text{Prob}(I \leq 0) = \text{Prob}(u \geq S) = \bar{F}(S) = \rho^S . \quad (4)$$

The expected amount of backorders is:

$$L(\lambda, \mu, S) = \sum_{w=S}^{\infty} (w - S) P_w = \frac{\rho^{S+1}}{1 - \rho} . \quad (5)$$

The expected inventory level, $I(\lambda, \mu, S)$, is given by:

$$I(\lambda, \mu, S) = \sum_{w=0}^S (S-w)P_w = S - \frac{\rho}{1-\rho}(1-\rho^S). \quad (6)$$

The producer and the supplier are both supposed risk neutral, i.e., their utility functions are the expected values of their profits. Production costs are supposed proportional to the quantity produced.

The expected profit rate of the producer is given by:

$$\begin{aligned} \pi_p(\lambda, \mu, S) &= (p - c_p)\lambda - hI(\lambda, \mu, S) - bL(\lambda, \mu, S) - T(\lambda, \mu, S) \\ &= (p - c_p)\lambda - (h + b)\frac{\rho^{S+1}}{1-\rho} - h\left(S - \frac{\rho}{1-\rho}\right) - T(\lambda, \mu, S). \end{aligned} \quad (7)$$

The supplier production costs involve a “fixed” part, related to the production capacity, characterized by the average production rate, μ , and a “variable” part, proportional to the expected sales rate. The expected profit rate of the supplier is given by:

$$\pi_s(\lambda, \mu, S) = T(\lambda, \mu, S) - c_v\lambda - c_\mu\mu. \quad (8)$$

As long as the stability condition $\mu > \lambda$ is satisfied, the global supply chain expected profit takes the form:

$$\begin{aligned} \Pi(\lambda, \mu, S) &= \pi_p(\lambda, \mu, S) + \pi_s(\lambda, \mu, S) \\ &= (p - c_p - c_v)\lambda - c_\mu\mu - (h + b)\frac{\rho^{S+1}}{1-\rho} - h\left(S - \frac{\rho}{1-\rho}\right). \end{aligned} \quad (9)$$

For given values of λ , μ and p , the optimal inventory quantity for the global supply chain, S_G^* , is unique and satisfies the classical discrete newsvendor formula for the M/M/1 case ([2], [8]):

$$S_G^* = \left\lfloor \frac{\text{Log} \frac{h}{h+b}}{\text{Log}(\rho)} \right\rfloor \quad (10)$$

where $\lfloor \cdot \rfloor$ stands for the integral part of a real quantity.

It can be noted that if the transfer function $T(\lambda, \mu, S)$ does not depend on S , as it is the case in “price-only” contracts, the optimal value S_G^* also maximizes $\pi_p(\lambda, \mu, S)$ since the inventory is completely owned by the producer. On the contrary, the value of S becomes irrelevant to the supplier.

In the opposite case, when $T(\lambda, \mu, S)$ does depend on S , decisional autonomy of both the supplier and the producer may shift the total inventory quantity to a suboptimal value.

Another question of interest in selecting a transfer payment contract is the share of the global profit obtained by each partner. A game-theoretic approach may help determining an appropriate transfer function $T(\lambda, \mu, S)$ such that the global optimal profit could be obtained with a fair distribution of profit among the partners.

3 A game-theory approach to supply chain coordination

3.1 A game theory model of a decision centre

The enterprises of a supply chain share a common goal: to hold a share in a market. They join their resources to produce and sell particular goods. So, their main factor of integration is the production process for these goods but they have different constraints and, beyond their common goal, their objectives – say profit maximization– may be conflicting and even antagonistic. Cooperative game theory can be of great help to design a supply chain or a virtual enterprise by selecting an optimal coalition of partners. But a non-cooperative (also called strategic) approach is certainly more appropriate to determine the set of equilibrium points that can be reached in trade conditions. A case of particular interest is when there exist decisional states from which neither player has interest to depart. Such cases, called Nash equilibria, have received a considerable attention both in theory and applications of Game Theory (see e.g. [9]). A particular property of a Nash equilibrium point is that, if the internal models of the players are known it is immediately reached at the first iteration of the game.

Therefore, existence of Nash equilibrium points reduces the negotiation process to a one-shot exchange of information. In many real situations, the equilibrium is not unique, and the first player imposes the outcome of the game. The particular equilibrium reached in such an asymmetric game is called "Stackelberg equilibrium".

Outside the full information context, the outcome of a game generally depends on who plays first and how the players negotiate. So far, the general case of sequential games with incomplete information has not received much attention in the literature.

3.2 Equilibrium states

The analysis naturally starts from the risk neutral situation in which the individual goal of each player is uniquely to maximize his/her gain or to minimize his/her loss. Consider the case of m decision variables, (q_1, \dots, q_m) and n players with utility functions $\pi_1(q_1, \dots, q_m), \dots, \pi_n(q_1, \dots, q_m)$. It can be assumed that each player i has some decision variables imposed (uncontrollable to him) and others that he can choose (set of controllable variables Γ_i).

An equilibrium state corresponds, for each player, to partial optimality with respect to his controllable variables :

$$\pi_i(q^*_1, \dots, q^*_j, \dots, q^*_m) \geq \pi_i(q^*_1, \dots, q_j, \dots, q^*_m) \forall q_j \in \Gamma_i \quad (11)$$

In the case of a continuous decision set, inequality (11) leads to a necessary condition of the following type:

$$\frac{\partial \pi_i}{\partial q_j}(q^*_1, \dots, q^*_j, \dots, q^*_m) = 0. \quad (12)$$

Additionally, each player has a minimal expectation on his/her utility function:

$$\pi_i(q^*_1, \dots, q^*_j, \dots, q^*_m) \geq \pi_i^0 \quad (13)$$

In practical cases, it soon appears that without a coordination mechanism, no agreement can be

obtained by negotiation since the set of constraints (11) and (13) is generally incompatible. As a consequence of this observation, the main purpose of a contract in our study will be to modify the utility functions of the players:

-- to generate a not empty feasible set of values Σ for (q_1, \dots, q_m) that satisfies the set of constraints (13)

-- to identify a point $(q^*_1, \dots, q^*_j, \dots, q^*_m) \in \Sigma$ that satisfies the optimality conditions (11).

Furthermore, a successful negotiation requires not only the non-emptiness of Σ , but also its reachability with convergence to $(q^*_1, \dots, q^*_j, \dots, q^*_m)$

3.3 *Coordination through contracts*

A contract can be characterized by several parameters that strengthen the commitments of partners through risk, profit or cost sharing, and/or move the equilibrium state toward a better global performance. The values of contract parameters are generally determined through a negotiation process between the partners. According to [6], a coordinating contract is “one that results in a Pareto-optimal solution acceptable to each agent”. A contract negotiation results in a game generally involving a leader and a follower. Examples are capacity games ruled by the producer [3], cutoff policies decided by the supplier [5].

The purpose of this study is to determine the functions and parameters to be integrated in the utility functions of the actors so as to move their local optimal decision toward a mutually preferred equilibrium state.

4 **Decision integration through contracts**

4.1 *Coordination of decisions in an enterprise network*

The supply chains considered in this study have a distributed nature that results in heterogeneous

decision centers. An analysis is thus needed to identify possible sources of conflicts and to correct them through contracts. The viewpoint of each type of actor: customer, producer, supplier, will be characterized by a utility function and one specific decision variable. The customer will decide on the most acceptable value of the delivery time, the producer will chose the end-product inventory level and the supplier will set the production capacity. The optimal values of decision variables will be conditioned by some parameters that define the terms of the contracts. By indirectly controlling the decision, the actor who determines the contract parameters acts a Stackelberg leader, while the one who optimally selects the value of the decision variable on the basis of the contract actually acts as a Stackelberg follower. Along this scheme, several types of contracts will be proposed, to drive the system close to its global equilibrium, while maintaining the decentralized decisional structure. Such contracts are applied to the simple model presented above. However, they imply variables of decision and mechanisms of compensation which could easily apply to the real supply chains, subject to parametric adjustments obtained by simulation.

4.2 *The customer viewpoint*

For the customer, the utility function depends on the quality of the product that is both its added-value and its lead-time. Using the relations (3) and (4), the expected lead time of customer orders is defined by:

$$W = \text{Prob}(I \leq 0) \tau = \rho^S / (\mu - \lambda) = L / \lambda \quad (14)$$

Let a denote the unit value of the product and $\varphi(L)$ the cost function of backorders for the customer.

$$\pi_c(L) = (a - p)\lambda - \varphi(L) \quad (15)$$

In this section, the values of λ, μ, S are supposed exogenously given. Under the rational assumption that function $\varphi(L)$ is monotonously increasing in L , maximization of customer satisfaction reduces to minimizing his amount of backorders, L . Thus for the customer, the optimal value of L is 0. It is

obtained for $S \rightarrow \infty$ and/or $\mu \rightarrow \infty$. It is clear that such a customer utility function is not acceptable and even impossible to satisfy for the producer and the supplier.

A contract then has to be negotiated between the customer and the producer. Suppose now, from a realistic point of view, that $\varphi(L)$ is strictly convex in the interval $[0, \bar{L}]$, with $\frac{\partial \varphi}{\partial L}(0) = 0$. Then, the following result can be shown.

Proposition

Under a linear price adjustment mechanism (p_0, η) , there is a unique maximal value of the customer utility function in the interval $[0, \bar{L}]$.

Proof

Consider the linear price adjustment mechanism (p_0, η) :

$$p(L) = p_0 - \eta W = p_0 - \eta L / \lambda \quad \text{with } p_0 > 0, \eta > 0 \quad (16)$$

The new customer utility function is given by:

$$\pi'_c(L) = (a - p_0)\lambda + \eta L - \varphi(L) \quad (17)$$

Strict convexity of $\varphi(L)$ in the interval $[0, \bar{L}]$ implies strict concavity of $\pi'_c(L)$ in $[0, \bar{L}]$.

Furthermore, $\frac{\partial \pi'_c}{\partial L}(L) = \eta - \frac{\partial \varphi}{\partial L}(L)$ and $\frac{\partial \pi'_c}{\partial L}(0) > 0$. Then, by concavity, the value of $L^* \in [0, \bar{L}]$ for

which $\pi'_c(L)$ is maximal is unique. It is $L^* = \bar{L}$ if $\frac{\partial \pi'_c}{\partial L}(\bar{L}) \geq 0$ or uniquely defined by $\frac{\partial \pi'_c}{\partial L}(L^*) = 0$ if

$$\frac{\partial \pi'_c}{\partial L}(\bar{L}) < 0.$$

Notes:

It is natural to assume $\frac{\partial \pi'_c}{\partial L}(\bar{L}) < 0$ since for large values of L , the marginal inconvenience on lead-time dominates the marginal utility of price discount.

Two possible techniques can be used to implement this contract: either the price is constant but there is a penalty that depends on the average amount of backorders, or the price varies as a function of the observed lateness of delivery. Both cases correspond, on the average, to the contract described by (16).

In this section, the value of λ is supposed constant. However, a dependence on λ , $p_0(\lambda)$, is introduced in the sequel, to counteract a price increase if λ , and therefore L decrease. Such a price increase would tend to further decrease λ and therefore to be economically unproductive.

From the global supply chain viewpoint, the value of L^* maximizing the consumer utility function under the pricing mechanism (16), should also be globally optimal for the whole supply chain. Assuming

$\frac{\partial \pi'_c}{\partial L}(\bar{L}) < 0$, the choice of the discount coefficient, $\eta > 0$ should then satisfy :

$$\frac{\partial \pi'_c}{\partial L}(L^*) = 0 \Rightarrow \eta^* = \frac{\partial \varphi}{\partial L}(L^*). \quad (18)$$

The difficulty in this coordination mechanism is determination of the marginal increase of inconvenience, $\frac{\partial \varphi}{\partial L}(L^*)$.

In a supply-chain system, customers and producers determine the contract parameters through negotiation. In the case that the internal models of the partners are their private information, the value of the lead-time compensation η is determined through a bargaining game of alternating offers, in which the players alternate making offers until one is accepted. To describe this kind of a game, we need to introduce the variable "time". The first move of the game occurs in period 0, when the producer makes a proposal for the value η and asks for an answer concerning the amount of backorders $L^*(\eta)$ that the customer is willing to select under contract (p_0, η) . In period 1, the customer makes a proposal by

announcing his optimal amount of backorders $L^*(\eta)$, and the corresponding price, $p(L^*(\eta))$ which he is ready to pay. The producer may accept or reject this proposal in period 2. Acceptance ends the game while rejection leads to a new proposal in which the producer adjusts the value of η and proposes an updated contract (p_0, η) . The game continues in this fashion until an agreement or, if no offer is ever accepted, until the disagreement event. We note that the partners prefer an agreement rather than the disagreement event and they seek to reach an agreement as soon as possible since time is valuable. Under these assumptions, the bargaining game of the partners can be modeled as an extensive game with perfect information. It can reach a Nash equilibrium that is defined using the dependence (5) of L on (λ, μ, S) . In the equilibrium, the optimal value of the lead-time compensation η^* should satisfy:

$$L^*(\eta^*) = \frac{\rho^{(S^*(\eta^*)+1)}}{1-\rho} \quad (19)$$

To illustrate this section, an example of inconvenience function $\varphi(L) = 5 + \frac{10}{\pi} \text{Arc tan}(\frac{L-10}{2})$ has been selected. Fig.2 represents the evolution with L (in the range $[0, \bar{L}]$ with $\bar{L} = 10$) of the customer utility function under fixed price $p = 8.2$ (2a) and under the pricing mechanism (2b), with $\lambda = 1, p_0(1) = 8.5, a = 15, \eta = 0.5$.

In this example, the optimal value of L for the customer is: $L^* \approx 7$.

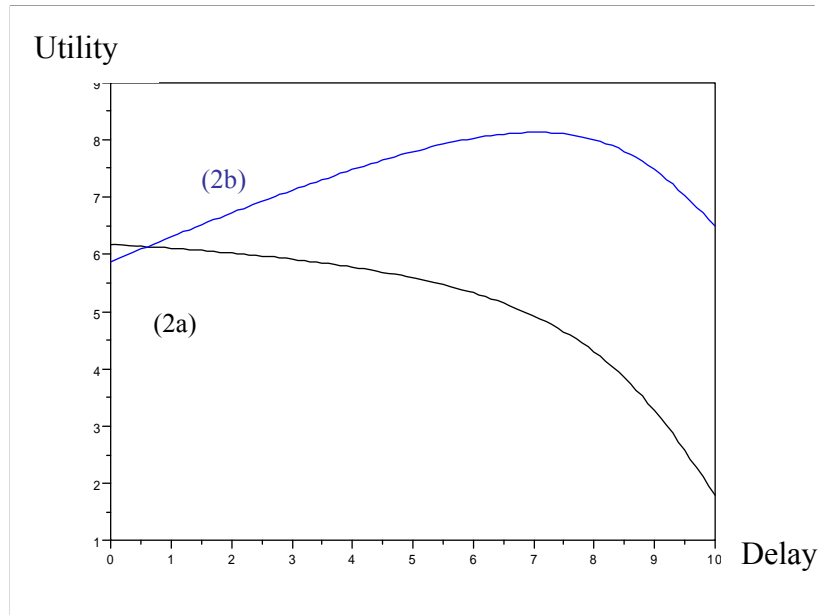


Fig.2 Customer utility function with fixed price (2a) and delay dependent price (2b)

4.3 The producer viewpoint

Assume that λ has been determined by the market, as a result of the contract $[p_0(\lambda), \eta]$ agreed by the customer and the producer. Then, the supplier decision variable μ is determined by negotiation with the producer, and the internal optimization problem of the producer reduces to selecting her optimal inventory level, S^* . Introduction of (16) in the expression of $\pi_p(\lambda, \mu, S)$ yields:

$$\pi_p(\lambda, \mu, S) = [p_0(\lambda) - c_p] \lambda - (h + b + \eta) \frac{\rho^{S+1}}{1 - \rho} - h \left(S - \frac{\rho}{1 - \rho} \right) - T(\lambda, \mu, S).$$

Under a price-only contract, the payment transfer $T(\lambda, \mu, S)$ does not depend on S . Then, the producer profit function is concave in S and the optimal inventory capacity is given by the classical newsvendor formula:

$$S^*(\eta) = \left\lceil \frac{\text{Log} \frac{h}{h + b + \eta}}{\text{Log}(\rho)} \right\rceil = \left\lceil \frac{\text{Log}(h + b + \eta) - \text{Log}(h)}{\text{Log}(\mu) - \text{Log}(\lambda)} \right\rceil \quad (20)$$

According to formula (20), S^* is a decreasing function of μ , as shown on Fig.3. Thus, as the

production capacity of the supplier increases, the producer can reduce her optimal inventory level S^* .

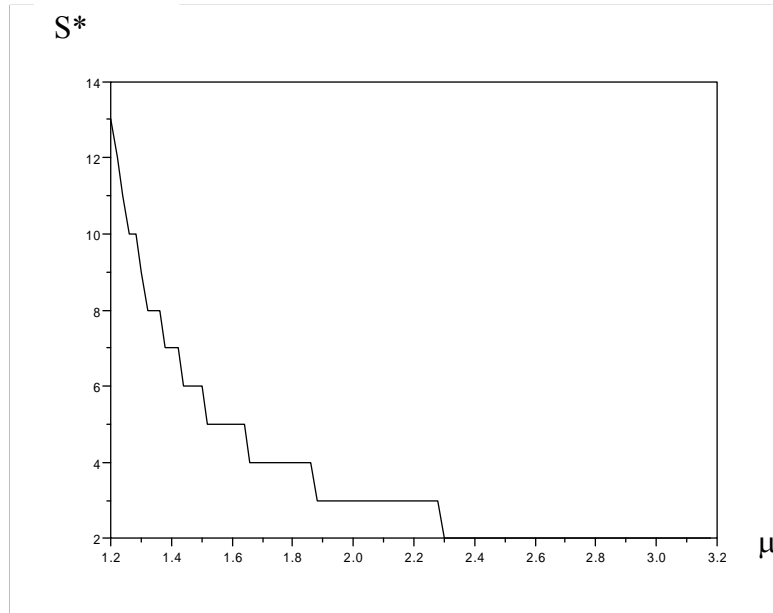


Fig.3 Evolution of S^* with μ ($\lambda=1, h=0.5, b=5, \eta=0.5$)

4.4 The supplier viewpoint

If we now assume that λ is given, that L takes the value L^* , determined by the customer, and S the value S^* determined by the producer, then the supplier should reserve the capacity to assure supply with rate μ^* determined to optimize $\pi_s(\lambda, \mu, S)$. From expression (8), it clearly appears that the nature of the transfer payment $T(\lambda, \mu, S)$ is essential in maximization of the supplier utility function. In the case of a delay independent price, the capacity to be installed by the supplier would tend to be λ , which is clearly unacceptable by the customer (Fig.1) and by the producer (Fig.2). A delay-dependent transfer payment will then be considered. As in [4], it seems appropriate to select a transfer payment linearly decreasing with the delivery delay, τ . This assumption defines a contract (r, k) with delivery delay dependent price. Parameter k is to be decided by the producer for a given reference price, r . From (3) and with k as a scaling coefficient, the average transfer takes the value:

$$T(\lambda, \mu, S) = \left(r - \frac{k}{\mu - \lambda}\right)\lambda \quad (21)$$

Under this contract, the derivative of the supplier profit function with respect to μ is:

$$\frac{\partial \pi_S}{\partial \mu} = \frac{k\lambda}{(\mu - \lambda)^2} - c_\mu$$

Therefore, for $\mu > \lambda$, the supplier profit function is concave and reaches its maximal value for :

$$\mu^* = \lambda + \sqrt{\frac{k\lambda}{c_\mu}} \quad (22)$$

The transfer payment then takes the value:

$$T(\lambda, \mu^*, S) = (r - \sqrt{\frac{kc_\mu}{\lambda}})\lambda \quad (23)$$

Another constraint to be satisfied is positivity of $\pi_s(\lambda, \mu, S)$, which requires:

$$r \geq c_v + c_\mu + 2\sqrt{\frac{kc_\mu}{\lambda}} \quad (24)$$

Under contract (r, k) the transfer payment is independent of S . The (supplier-producer) system can then be coordinated by the producer who acts as the Stackelberg leader by selecting parameter k for a given price r for which the supplier, if he acts rationally, will install the capacity μ^* given by (22). Note that any stabilizing value of μ^* ($\mu^* > \lambda$) can be obtained by (22) through an appropriate choice of k . Also, the optimal value of S , S^* given by (20) can be chosen by the producer without any negotiation. But then, the profit optimizing producer imposes the largest value of μ^* acceptable by the supplier, to minimize her reference inventory level, S^* and the associated inventory costs. The limit border to the pair of values (r, k) accepted by the supplier is obtained for the minimal acceptable profit, $\underline{\pi}_s$. In particular, condition (24) corresponds to $\underline{\pi}_s = 0$. Evolution of profit values with μ are shown in Fig.4.

For the numerical values of Fig.2, the producer reference inventory level is: $S^*=6$. Condition (24) then

gives the maximal value of k acceptable by the supplier for $r=3$ and $c_\mu = c_v=1$: $k=0.25$. For the choice $k=0.25$ imposed by the producer, the optimal value of μ ($\mu^*=1.5$) is obtained from (22).

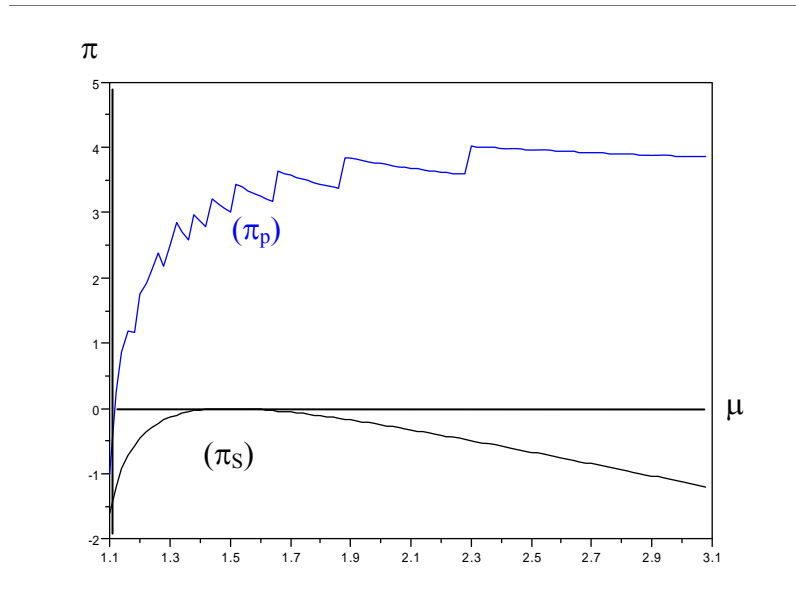


Fig.4 Supplier and producer profit curves

5 Conclusions

In enterprise networks, decentralized decisions are generally less efficient than a centralized mechanism maximizing a global utility function. In particular, when decisions are decentralized with different utility functions, a system with a dominant actor usually leads to a Stackelberg game in which the leader gets the maximal value of his/her utility function while the followers are maintained at their minimal acceptable satisfaction level. However, in spite of this unbalance, such an equilibrium may correspond to global optimal conditions, provided that the contracts between the partners allow for a shift of local equilibria toward globally optimal values. In this respect, different contracts have been shown to possess this property: delay adjustment mechanisms for retail prices, delay dependent prices for delivery from the supplier.

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