Production Coordination in a Multistage Manufacturing Network

Jean-Claude Hennet
CNRS, LISA, 62 avenue Notre Dame du Lac
49000 Angers, France

Abstract:
The study describes a model of a network of enterprises that cooperate to manufacture end products from raw materials. The product structure determines the organisation of the multistage supply chain. Production control policies are proposed to achieve production synchronization under the assumption of random demands for end products, and using inventories for components and end products. It is shown that a promising technique to limit production fluctuations at each stage is to combine a base stock policy with a distributed MRP computation from expected demands for end products. A critical factor for an efficient implementation of this combined policy is to smoothly reach the steady state multistage production regime.

Keywords:
Extended Enterprise, Manufacturing Networks, Supply Chain Integration, MRP, Base-Stock.

1. Introduction

This study considers a multistage production system with several producers. In contrast to a multistage system owned by a single producer, such a system is characterized by its distributed nature, which entails the difficulty to efficiently implement onto the whole system, integrated planning software such as ERP and APS. Networks of enterprises are typically characterized by possible conflicts of interest, randomness and uncertainty of lead-times and transportation delays, control actions with limited impacts. However, they also have advantages in terms of flexibility, plasticity (easy reconfiguration possibilities) and a quick adaptation to the market, in terms of quality and quantity. Ideally, a network structure can be designed in a perfect adequation with the identified needs and requirements of a product, a project or a process.

Multistage production is a good challenge to motivate the design of a supply chain as a virtual enterprise, by gathering in an enterprise network the most effective enterprises for each production stage. In a multistage production network, the product structure dictates the organization: producers of primary products play the role of suppliers, producers of intermediate products play both roles of suppliers and producers, and producers of end products are both producers and retailers. Supply chain design should be optimized not only through selection of the best coalition, negotiation for task assignment and business contracts, but also through organization of information and product flows. Many studies have shown that local effectiveness is far from automatically inducing the global efficiency of the whole production process. Several authors have proposed contracts to improve global efficiency while respecting the decisional autonomy of the partners (see e.g. [Cachon, and Zipkin. 1999], [Caldentey, and Wein, 2003]).

The study aims at extending to a multistage supply chain, the bimodal planning scheme proposed in [Hennet, 2003]. The time horizon is decomposed into two parts. The long-term plan implements a robust state feedback in a region around the optimal running conditions, while the short-term part drives the system to this region while meeting the demand in the most economical manner.

Steady-state running conditions correspond to a permanent regime under stationary demands for end products. Such a regime can be characterized at each stage of the supply chain by a control that regulates the system deviations from nominal production and inventory levels. Changes in mean demand rates can be absorbed if their amplitudes are limited. Beyond these limits, a new permanent regime and a new transient trajectory have to be computed. In the case of a distributed multistage system, an additional difficulty is the combination of a high level of interaction between product flows and a low level of decisional coordination during the transient phase. The marketing of new products or important changes in consumers tastes generate large deviations on stocks and production levels that tend to amplify upward the supply chain. This phenomenon, largely described and studied in the supply chain literature, is often referred to as the "bullwhip effect". It has been recognized as a control problem, not solved by classical ordering policies such as the (s,S) policy, unable to attenuate the disturbances and damp the oscillations [Dejonckheere et al., 2003]. In an integrated decisional framework, ordering policies based on (filtered) final demands and inventory positions (rather than on inventory levels) perform remarkably well. Their application to distributed enterprise networks is often jeopardized by the lack of advanced information on final demands at early stages of the process and by the conservativeness of local production and inventory control policies. It is rather proposed to perform frequent adjustments to customers requirements and to design transient...
trajectories so as to smoothly reach robust and efficient running conditions.

2. The model

2.1. Assumptions

The models considered in this study are planning models over a medium-term and a long-term time horizon. In this framework, time is generally discretized in large buckets which can be of equal or unequal duration. In [Henmet, 2003], a time decomposition of the horizon has been proposed, mainly to take into account two different information patterns on external demand: in the short-term, orders are firm and precisely known, and after the short-term time horizon, they are uncertain or random and can be modelled by probabilistic distributions. Such a decomposition is selected in this study, but an additional factor of complexity is introduced by considering a production network with an imperfect transfer of information along the supply chain.

The considered manufacturing networks are multi-stage. With constant lead-times classically used in MRP-based approaches.

2.2. The multistage model

The manufacturing model is based on the multistage structure of products. It uses the assumption of constant lead times classically used in MRP-based approaches.

After the short term time horizon, the vector of external demands is decomposed into a predicted component and a disturbance:

\[ d_{t} = \hat{d}_{t} + e_{t} \]  

with \( d_{t,1} = 0 \) for primary and intermediate products \( (i=n_{e}+1,...,N) \), and \( e_{i,t} \) has mean value 0 and standard variation \( \sigma_{i} \) for end products \( (i \in \{1,n_{e}\} \).

Demand disturbances for end products are known several periods in advance by end producers but unpredictable for primary and intermediate manufacturers: \( \hat{d}_{i,t} = \hat{d}_{i,t+k} \) for \( k \geq 1, i \in \{1,n_{e}\} \).

Due to non negativity of demand sequences \( (d_{i,t}) \), disturbances on orders for end products can be assumed bounded from below. It is also realistic to assume them bounded from above. Then, if \( i \) is an end product \( (i \in \{1,n_{e}\} \),

\[ -\omega_{i} \leq e_{i,t} \leq \omega_{i}, \text{ with } \omega_{i} \geq 0, \omega_{i} \geq 0, i \in \{1,n_{e}\} \]  

If the probability distribution of \( e_{i,t} \) is uniform in \((-\omega_{i},\omega_{i})\), then its mean standard deviation is:

\[ \sigma_{i} = \frac{\omega_{i}}{\sqrt{3}} \]  

In the sequel, demands for end products at each period will be supposed uniformly distributed on \([\hat{d}_{i,t} - \omega_{i}, \hat{d}_{i,t} + \omega_{i}]\).

The assumption of constant lead times benefits the assumption of linear multivariable model with distributed delays and capacity constraints on inventories and production.

The system output matrix of the system has dimension \( N \times N \). It is defined by:

\[ \text{Diag}(z^{-\theta}) = \begin{bmatrix} z^{-\theta_{1}} & 0 & \cdots & 0 \\ 0 & z^{-\theta_{2}} & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & z^{-\theta_{N}} \end{bmatrix} \]  

The input matrix is also \( N \times N \). It is defined by:

\[ \Pi = \begin{bmatrix} \pi_{21} & \cdots & \cdot & \cdots & 0 \\ \cdot & \cdots & \ddots & \cdots & \vdots \\ \cdot & \cdots & \cdots & \pi_{N,N-1} & 0 \end{bmatrix} \]  

The input-output system equation takes the form:

\[ s_{t} = s_{t-1} + [\text{Diag}(z^{-\theta}) - \Pi] u_{t} - d_{t} \]  

under non negativity and capacity constraints:

\[ u_{t} \geq 0, s_{t} \geq 0 \text{ for } i=n_{e}+1,...,N. \]  

If backorders are allowed for end products, they can be represented, classically, as negative stocks. It is
important to note that non negativity of inventory variables for components is an essential logical condition that determines feasibility of a production vector \( u_t \).

Let \( n_p^R \) denote the amount of stock \( p \) needed to store one unit of product \( i \). For each of the \( P \) storage zones, located in the different enterprises, inventory capacity constraints take the following form:

\[
\sum_{i=1}^{N} n_p^R s_{i,t} \leq N_p
\]

Let \( m_i^r \) denote the amount of resource \( r \) needed per period to produce one unit of product \( i \). It is supposed that the same amount of resource is required during the lead time. Production constraints then take the following form:

\[
\sum_{i=1}^{N} \sum_{r=1}^{R} m_i^r u_{i,t-1} \leq M_r \quad \text{for} \quad r=1,...,R, t=1,...
\]

3. Production control policies

3.1. Production and Inventories

The considered production control policies combine the base stock approach, to maintain constant the inventory positions in stationary conditions, and the MRP approach to order components and manufacture products in agreement with the expected demand for end products. Exact demands are not supposed to be known in advance, except at the last manufacturing stage. Advance information is filtered by end producers and not propagated upward the supply chain. In order to limit production fluctuations due to random demands and the so-called bullwhip effect, manufacturers base their production policy on statistically estimated mean demands rather than on the actual orders for components received from their direct customer. Such a policy combines open-loop with closed-loop properties. It regulates the system toward reference levels for production and inventory at each stage of the supply chain.

3.2. A distributed multistage basestock control policy

In the proposed combined policy, the vector of production levels \( u_t = (u_{i,j}) \) is decomposed into two parts:

\[
u_t = u_t^* + \nu_t
\]

where

- \( u_t^* \) is the vector of gross production levels
- \( \nu_t \) the vector of inventory replenishment under the base stock policy
- \( s^* \) vector of nominal safety stocks levels, supposed constant over the considered time horizon. According to the selected base-stock policy, the vector of incremental stock level at period \( t \) is defined by:

\[
y_t = s_t - s^* 
\]

In stationary conditions, \( s_{t+1} = s_t = s^* \). Then, equation (5) implies:

\[
T(z)u_t^* = \delta_t
\]

with \( T(z) = [\text{Diag}(-\theta_i^T) - \Pi] \).

Matrix \( T(z) \) is decomposed as follows:

\[
T(z) = T_0 + z^{-1}T_1 + \cdots + z^{-\theta}T_\theta
\]

Note that if \( \theta_i \geq 1 \) \( \forall i = 1,...,N \) then, \( T_0 = -\Pi \) and \( T_1 + \cdots + T_\theta = I \).

Due to the structure and delay terms of matrix \( T(z) \), \( T^{-1}(z) \) is strictly anticipative. If capacity constraints (7), (8) are satisfied by this solution, condition (11) may thus be achieved by:

\[
u_t^* = T^{-1}(z)\delta_t
\]

Let \( \delta \) be the maximal degree in \( z \) of polynomial matrix \( T^{-1}(z) \). One can write: \( T^{-1}(z) = z^{\delta}Q(z) \) where matrix \( Q(z) \), which is polynomial in \( z^{-1} \), is decomposed as follows:

\[
Q(z) = Q_0 + z^{-1}Q_1 + \cdots + z^{-\delta}Q_{\delta}.
\]

It can be noted that, due to the structure of \( T(z) \), matrices \( Q_i, \ i \in [1,\delta] \) are lower triangular with nonnegative coefficient. The system delay, \( \delta \) then corresponds to the longest path time in the product structure graph from raw materials to end products.

Equation (15) can then be re-written:

\[
u_t^* = \sum_{k=0}^{\delta} Q_k \delta_{t+k}
\]

The vector of nominal safety stock levels is \( s^* = (s_i^*) \). In a decentralized production network, it is natural to assume that each component of this vector is computed independently. Then, using Hadley-Whitin’s approximation [Hadley and Whitin, 1963] of inventory levels for each product \( i=1,...,n \), \( s_i^* \) can be classically chosen so as to guarantee some quality bound on the stockout probability:

\[
\text{Prob}(\text{supplemental demand during} \ t_i \leq s_i^*) \geq 1 - \epsilon_i.
\]

The no stockout limit probabilities can be achieved with basestock levels given by the classical newsboy formula (see e.g. Porteus, 1990):
\[ s_i = \Phi_i^{-1}(1 - e_i) \] with \( e_i = \frac{h_i + c_i}{h_i + b_i} \) \hspace{1cm} (16)

where \( \Phi_i(\cdot) \) is the cumulated PDF of supplemental demand for product \( i \) during lead time \( \theta_i \), \( c_i \) is the unit production cost for product \( i \), \( h_i \) is the unit holding cost per time unit for product \( i \), \( b_i \) is the unit backorder cost per time unit for product \( i \).

Under uniformly distributed demand disturbances for final products, \( \Phi_i(q) = \frac{1}{2} + \frac{q}{2a_i} \) with \( a_i = \theta_i [I - \Pi]^{-1} \Omega \), and thus: \( s_i = a_i \frac{2h_i - h_i - 2c_i}{h_i + b_i} \).

### 3.3. An integrated predictive MRP policy

Using (1), (5), (9), (10) and (11), the system equation is rewritten as follows:

\[(1 - z^{-1})y_t = T(z) v_t - e_t \] \hspace{1cm} (17)

With \( \theta = \max \theta_i \), (17) leads to the predictive equation:

\[ \hat{y}_{t+\theta} = y_{t-1} + U_0 v_t + U_1 v_{t-1} + \ldots + U_{\theta} v_{t-\theta} \] \hspace{1cm} (18)

with matrices \( U_i \) defined by: \( U_i = \sum_{j=1}^{\theta} T_j \).

A predictively control law can then be obtained by setting: \( \hat{y}_{t+\theta} = 0 \) in (18).

In an integrated framework, as the one studied in [Hennet 2003], the inventory replenishment control can take the form of a state feedback:

\[ v_t = F y_{t-1} + \sum_{l=1}^{\theta} G_l v_{t-1} \] \hspace{1cm} (19)

with \( F = -(I - \Pi) \), \( G_l = -(I - \Pi)^{-1} U_i \), \( i = 1, \ldots, n \).

The production vector would then satisfy:

\[ u_t = Q(z) \hat{d}_{t+\delta} + F(s_{t-1} - s^* + \sum_{l=1}^{\theta} G_l v_{t-1}) \] \hspace{1cm} (20)

and the closed-loop system could then be written:

\[ s_t = s_{t-1} + T(z) v_t - e_t \] \hspace{1cm} (21)

Some advantages of the control law (19) are optimality, robustness and constraint satisfaction under some bounding conditions on disturbances [Hennet 2003]. However, equation (20) does not seem realistic for distributed production networks, because it involves, for each stage, all the information on stocks, production and demand from downstream stages. An important remark then has to be made. The feedback part of equation (20) does not merely apply the basestock feedback policy to the vector of inventory levels, \( s_{t-1} \), but rather to the vector of inventory positions, \( p_{t-1} \), each inventory position being classically defined as the sum of the inventory level and pending orders for product \( i \):

\[ p_{t, i-1} = s_{t, i-1} + \sum_{k=1}^{\theta} v_{t-k} \] \hspace{1cm} (22)

Policy (20) can then be reformulated as follows:

\[ u_t = Q(z) \hat{d}_{t+\delta} - (I - \Pi)^{-1}(p_{t-1} - s^*) \] \hspace{1cm} (23)

On the basis of this new formulation, a similar policy can now be defined to take into account the decentralized nature of the control vector.

### 3.4. A distributed MRP Policy

In the decentralized framework, each production unit associated to product \( i \) \((i \in \{1, 2, \ldots, N\})\) is supposed to have information on local inventory and production variables (\( s_{i,t}, u_{i,t-k}, k \geq 1 \)), and on demand predictions for end products: \( \hat{d}_{t+k}, k \in \{1, \delta\} \). Additionally, the production unit receives orders from downstream stages early enough to deliver products on time. Under these assumptions, the multistage production process may be coordinated through the choice of similar – but locally defined – production policies that combine a predictive part and a basestock feedback term on the inventory position: \( u_{it} = Q^i(z) \hat{d}_{t+\delta} + v_{it} \) with

\[ v_{it} = s_{i,t} - p_{i,t-1} + r_{it} \] \hspace{1cm} (24)

where \( Q^i(z) \) denotes the \( i \)-th row of \( Q(z) \), and \( r_{it} \) the total additional (replenishment) order for product \( i \) at period \( t \) coming from downstream production units: \( r_{it} = \sum_{j=1}^{N} \pi_{yj} v_{jt} \). This expression, together with (24), leads to expression (23).

And therefore, the integrated predictive MRP policy described in the previous section can be implemented in a distributed manner, as the sum of a predictive term (\( Q^i(z) \hat{d}_{t+\delta} \)), an order-based term (\( r_{it} \)), and a basestock inventory position regulation term:

\[ u_{it} = Q^i(z) \hat{d}_{t+\delta} + s_{i,t} - p_{i,t-1} + r_{it} \] \hspace{1cm} (25)

### 3.5. Toward a smooth accessibility to the target state

The first objective of the short term control policy is to drive the system to a target state from which the distributed MRP policy (25) can be implemented with stability and robustness. In this short-term transient phase, no external demand is served \((d(t) = 0)\) and consequently, the system is fully deterministic. The second control objective is to minimize the duration of this phase, so that the
system gets ready to serve the demand as soon as possible in optimal running conditions.

From expression (14), the minimal duration of the transient phase is \( \delta = \deg(T^{-1}(z)) \). The system is supposed to be initialized at period 0 with all previous variables null: \( u(t)=0, s(t)=0 \) for \( t \leq 0 \). Assuming that capacity constraints allow the system to reach its nominal regime in the minimal number of periods, \( \delta \), the target state can be characterized by \( t \leq \delta \).

Additionally, base stocks should be built for all the products, to obtain \( s_{\delta+1} = s_\delta = s^* \).

The just-in-time minimal time trajectory is then characterized by:

\[
  u_t = \hat{Q}_{\delta-t}s^* + \sum_{k=1}^{\delta} \hat{Q}_{\delta+k-t} \hat{d}_{\delta+k} \quad \text{for } t=1, \ldots, \delta \quad (26)
\]

The first term in expression (26) corresponds to the building of the initial inventory. The associated production components may be anticipated for smoothing production. Such a smoothing mechanism, similar to the one often used in MRP II, facilitates satisfaction of production capacity constraints.

4. **An example**

![Diagram of a 5-product multistage example](EMSS2006)

Fig.1 A 5-product multistage example

In this example, the minimal duration of the transient phase is \( \delta = 5 \). Transient trajectories are computed from expression (26), using the smoothing mechanism, and steady state trajectories for production and inventory variables are computed by expressions (25). Simulations are run analytically with MATLAB, for uniformly distributed demands per period, in \( \{10,30\} \) for product 1 and in \( \{5,25\} \) for product 2. Figure 2 shows the evolution of production and stock levels under these random stationary demand patterns.
Conclusions

During the last fifteen years, many sophisticated software for integrated data processing and production planning have appeared on the market and have transformed the organization of most world class manufacturing companies. However, the emergence of production networks of heterogeneous enterprises call for new types of coordination tools, based on agreements and contracts rather than on vertical integration. In this new context that combines common goals with distributed decisions, classical production management issues such as planning, lot-sizing, inventory control and scheduling, have to be revisited to take this duality into account. In particular, the paper has shown that adaptation of the MRP approach to production networks leads to taking production and inventory decisions under limited information on the whole system. Furthermore, frequent adjustments to different market conditions draw a special attention onto transient trajectory from the initial state of a production unit to its optimal running conditions in a given economical context.

Partners should have contractual agreements and efficient communication and negotiation protocols.

Production planning should take into account the randomness of demands from customers. The role of stocks is then essential to compensate for uncertainties, but their level should be kept low enough through scheduling optimization to achieve a good synchronization of supply, production and sales.

References


