KEYWORDS
Ordering policies, bullwhip effect, randomness, multistage production.

ABSTRACT
The paper analyzes multi-stage supply chains subject to random variations of demands for final products. It is shown by analytical simulations that under classical distributed production and ordering policies, reactivity to demand changes generates important load and inventory fluctuations. This phenomenon, commonly called the “bullwhip effect” appears to be highly dependent on these policies. A new policy, called the “mean-demand driven policy” is proposed to attenuate this phenomenon.

INTRODUCTION
During the last fifteen years, supply chain analysis has become a major concern both in manufacturing theory and in industrial practice. In the extended sense, which is now prevailing in the literature, a supply chain associates all the enterprises that contribute to production and sale of a family of products (goods or services). In this view, an ideal supply chain can be seen as a virtual enterprise designed to satisfy some consumers needs in the most efficient and profitable way. Several performance indices have been proposed to evaluate the quality of a supply chain, particularly in terms of costs and value, decisional integration, agility, reactivity and reliability. Some obstacles to performance maximization have then appeared in the very nature of supply chains. In particular, the decisional autonomy of enterprises sets some limits to communication, coordination and integration between the interacting entities.

Responsiveness and reactivity to demand changes may generate fluctuations in work load and inventory levels that may be amplified as they propagate upward along a supply chain. Since the work of H. Lee [Lee et al. 1997], such a phenomenon has been referred to as the Bullwhip Effect. Observation of real industrial cases and simulation studies have shown examples of bullwhip effects. The bullwhip effect has been observed in many real supply chains, ranging from mechanical industries to several food sectors (see [Miragliotta, 2004]). A case reported by the “Supply Chain Simulation Workgroup” of Santa Fe Institute Business Network concerns Proctor and Gamble's Pampers division, who found “huge swings in weekly demand and orders within their supply chain”. Managers, researchers, engineers and students all over the world have been taught the bullwhip effect by playing the popular “Beer Game”, a simulation developed by John Sterman’s group at MIT’s Sloan School of Management [Sterman 1989]. The players of the game represent four nodes in an idealized supply chain for a Beer company: retailer, wholesaler, distributor, factory. Play of the game produced volatility much like that observed in real supply chains. As the backlog for orders increases, players order too much input products, forcing their suppliers into severe backlogs. Conversely, excessive production propagates downward along the supply chain and the decrease of orders amplifies.

Key factors for generating bullwhip effects are demand uncertainty and distributed delays, but it has been shown that the loading and ordering policies themselves tend to amplify the phenomenon, either by lack of global information or by not using properly the global information available. In particular, [Sterman, 1989] has stressed the destabilizing influence of misperception of the multiple feedback loops in the case of local base-stock policies.

In terms of control system theory, load and inventory variables can be viewed as the outputs of the supply chain system, the final demands being the external inputs subject to disturbances, and replenishment policies producing the controlled inputs. From this approach, several authors [Dejonckheere et al. 2003] have studied the stability property of the system under classical base-stock policies and they have proposed in particular to reduce the nervousness of this policy by using a proportional controller.

The boundary of the control action set for a supply chain system is determined by its distributed structure. [Miragliotta, 2004] has proposed a partitioning of the causes of the bullwhip effect into structural determinants and external triggers. More generically, this paper distinguishes internal and external factors and evaluates the possible improvements that can be expected from using relevant external data as parameters in local ordering and loading policies.

In spite of their will to cooperate, the partners of a supply chain generally have different constraints, objectives and practices. Local stock levels and production loads are
private data that can only be integrated in local ordering policies. However, with the progress of integrated information platforms, other data can be shared by all the partners of a supply chain. In this study, it is assumed that demands for the final products of the supply chain are communicated in real time to all the partners. Then, three types of loading and ordering policies will be studied: inventory-based local policies, internal orders-based policies and external orders based policies. This study will be conducted by analytical simulation with Scilab [Gomez, 1999] on a 5 products – 3 levels manufacturing network presented and modeled in section 2. The different ordering policies will be described in section 3. Then, section 4 will compare the simulation results and some conclusions will be finally presented.

A MODEL OF A SUPPLY CHAIN SYSTEM

Presentation of the dynamic model

The model proposed to describe a supply chain is similar to the dynamic system presented in [Hennet, 2003]. It is based on the product structure of the final products with the bills of materials and lead-times associated to manufacturing stages. Transportation times are supposed included in lead-times.

Classically (see e.g. [Muckstadt and Roundy 1993] and the references therein), a multistage production structure can be represented by an acyclic directed graph with nodes representing the production activities and arcs linking components to products. The total number of product types is \( n \). In the considered multistage production structures, there is a one-to-one correspondence between products and activities: each production activity has several input products and one output product. Production of one unit of product of type \( i \) \((i=1,\ldots,n)\) requires the assembly of components \( j=1,\ldots,n \) in quantities \( P_{ji} \). Products can be partitioned into levels. Level 0 is for end products, level \( l \) for products which are components of products of levels strictly less than \( l \), and of at least of one level \( l\)-1 product-type. If products are numbered in the increasing order of their level, matrix \( P=(P_{ji}) \) has a lower triangular structure with zeros on the diagonal.

Each enterprise of the supply chain is supposed to perform one or several production activities using components provided by other enterprises upward in the chain and/or providing components to other enterprises downward along the chain. As in [Hennet, 2003], the planning horizon \([0,T]\) is divided in time periods, product lead-times are supposed constant and multiple of the elementary time-period.

The lead-time associated with product \( i \) is denoted \( \theta_i \). In the “gozinto” graph, constant durations \( \theta_i \) are thus associated with nodes \( i \in (1,\ldots,n) \), and arcs \((i,j)\) are valued by \( P_{ij} \) to represent the “gozinto” coefficients of the products.

External demands for final products are supposed random with constant mean values and uniformly distributed bounded variations.

The quantities involved in the model are described below for \( i \in (1,\ldots,n), k \in (0,\ldots,T) \):

- \( s_{ik} \) is the quantity of product \( i \) in stock at the beginning of period \( k \),
- \( v_{ik} \) is the quantity of product \( i \) whose production is started at period \( k \),
- \( z_{ik} \) is the quantity of product \( i \) delivered at period \( k \).

Demands for intermediate products are consistent with the input (or gozinto) matrix, \( P \), and with production levels:

\[
d_{ik} = \sum_{j=1}^{N} P_{ij} v_{jk}.
\]

Production levels, inventory levels and backorders are subject to pointwise-in-time constraints: positivity constraints and capacity constraints. In particular, if \( c_{ik} \) is the production capacity for production of \( i \) at period \( k \), the production order should satisfy:

\[
v_{ik} \leq c_{ik}.
\]

Customers and producers items being delivered on stock as much and as soon as possible, delivery variables are given by:

\[
z_{ik} = \min(d_{ik},s_{ik} + v_{i,k-\theta}).
\]

Some properties of the model

The product structures considered are characterized by an output matrix which is the identity matrix (activity loads are measured in units of output products) and an input matrix \( P \) which is square, with nonnegative coefficients, lower-triangular with zeros on the main diagonal. Consider the vector of mean external demands for all the products \( d = [d_1 \ldots d_n]^T \). This vector being assumed constant, stability requires that the mean production vector \( v = [v_1 \ldots v_n]^T \) is constant in steady state conditions and satisfies: \( d = (I-P)v \).

Then, it is not difficult to show that the inverse of matrix \((I-P)\) exists and is non negative. It uniquely determines the mean equilibrium production vector:
\[ v = (I - P)^{-1}d \]  \hspace{1cm} (4)

A vector of equilibrium inventory levels
\[ S = [S_1 \ldots S_n] \] can also be defined. Its computation depends not only on the probability distributions of external demands but also on the ordering policy used for each activity.

A necessary condition for the system stability is that the equilibrium state defined by the couple \((v,S)\) satisfies all the capacity constraints.

A 5-products example
An example of a product structure for 2 final products (numbered 1 and 2) is shown in Fig. 1. This is a three-level product structure with final products 1 and 2 at level 0, intermediate product 3 at level 1 and primary products 4 and 5 at level 2.

It is assumed that the 5 products are manufactured by 5 different enterprises. Production capacities at each period are given by:
\[ C = \begin{bmatrix} 75 & 60 & 250 & 740 & 380 \end{bmatrix}^T. \]

The vector of external demands is stationary with mean value
\[ d = \begin{bmatrix} 20 & 15 & 0 & 0 & 0 \end{bmatrix}^T. \]

The corresponding bill of material, given by equation (4), gives the mean production vector:
\[ v = \begin{bmatrix} 20 & 15 & 70 & 210 & 190 \end{bmatrix}^T \]  \hspace{1cm} (5)

At any period \(k\), \(d_{1k}\) is a uniformly distributed random variables in the interval [0,40] and \(d_{ik}\) is a uniformly distributed random variables in the interval [0,30]. Accordingly, equilibrium inventory levels have been computed and used to initialize the inventory trajectories when simulating the production network under the different policies.

THE PRODUCTION AND ORDERING POLICIES
An ordering policy is a rule that determines the amounts of components ordered to suppliers as a function of the available knowledge on the current state of the system. In a similar manner, a production policy is a rule that determines the amounts of products to be manufactures as a function of the available knowledge on the current state of the system.

Classically, it can be assumed that in a supply chain, each enterprise only knows its current and past manufacturing and supply orders, its current and past demands (from partners and from customers). Additionally, some anticipated knowledge of future orders for each enterprise will be assumed to construct the “order-based policy”. In this study, it is also assumed that current and past values of external demands for final products are available for all the partners of the supply chain. Such an assumption opens new possibilities for building production and ordering policies.

The three types of policies that will be compared are the inventory-based policy, the order-based policy and the demand-based policy.

The inventory-based policy
This policy acts more or less on the Kanban principle. At each production stage and at each period, the quantity delivered is given by expression (3). This delivery triggers production order for product \(i\): 
\[ u_{ik} = \min(z_{ik}, C_i). \]

Components availability conditions the production level for stage \(i\) through:
\[ v_{ik} = \min(u_{ik}, \frac{S_{ik}}{P_{ik}}). \]  \hspace{1cm} (6)

What is typical of this policy is that internal orders for primary and intermediate products \(j; P_{ik} \neq 0\) are created from the current production level of products \(i\) through the following relation:
\[ d_{jk} = \sum_{i=1}^{5} P_{ik} v_{ik} - \theta_i \]  \hspace{1cm} (7)

Such a policy relies much on current inventory levels since they limit delivery and production at each node of the production network. Such limitations propagate upwards with some delay through the Kanban mechanism described by expression (7).

The order-based policy
The order-based policy obeys a mechanism that combines “lot for lot” MRP and capacity constraints. Each enterprise provides products from its stock as long as the inventory level remains positive. Backorders are avoided by anticipating production for orders that cannot be satisfied.
from the stock. The ordering mechanism propagates upward taking into account the bill of materials and lead times.

**The mean demand-driven policy**

The mean demand-driven policy is a very simple local policy in which production orders are computed from the bill of material (4) and from the estimation of mean demands for final products:

\[ v_k = (I - P)^{-1} \hat{d}_k. \]  

(8)

Backorders may be generated when production capacity and/or availability of input components are not sufficient to execute these orders. Recursive estimators of mean demands are constructed by taking the average of real demand data on a given time window noted \( W \):

\[ \hat{d}_k = \frac{1}{W} \sum_{i=1}^{k} d_i \quad \hat{d}_{k-1} + \frac{1}{W} d_k \]  

(9)

It can be noted that due to recursive demand estimation, the mean demand-driven policy is adaptive and can be applied to non stationary demand profiles.

**NUMERICAL RESULTS**

In order to evaluate the bullwhip effects associated to the three policies, production and inventory variances for the 5 products have been estimated by series of 300 runs over a 40 period time horizon. To each run corresponds a random generation of uniformly distributed demand sequences for products 1 and 2, respectively in the intervals [0,40] and [0,30]. Then, model-based simulations have been performed using Scilab. [Gomez, 1999]. The results are presented in table 1.

Also, for comparison purposes, the same random demand trajectories of \((d_1)\) and \((d_2)\) have been used for the three policies. These demand trajectories are displayed on Fig. 2. Then, figures 3, 4 and 5 show the production and inventory evolutions for the 5 products under the three policies for data of Fig.2.

<table>
<thead>
<tr>
<th>Product</th>
<th>Production</th>
<th>Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Production 1" /></td>
<td><img src="image2" alt="Inventory 1" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image3" alt="Production 2" /></td>
<td><img src="image4" alt="Inventory 2" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image5" alt="Production 3" /></td>
<td><img src="image6" alt="Inventory 3" /></td>
</tr>
<tr>
<td>4</td>
<td><img src="image7" alt="Production 4" /></td>
<td><img src="image8" alt="Inventory 4" /></td>
</tr>
<tr>
<td>5</td>
<td><img src="image9" alt="Production 5" /></td>
<td><img src="image10" alt="Inventory 5" /></td>
</tr>
</tbody>
</table>

**Figure 3 Production and inventory levels**

From simulation results of Fig. 3, it can be noted that the inventory-based policy sustains important variations of product load and product inventory at all the production stages.

**The order-based policy**

As for the "lot for lot" MRP technique, the order-based policy uses the initial stock in the transient stage and then, stocks are maintained at the zero level, production anticipating demand. Better stability results have been obtained for this policy in terms of demand satisfaction. Compared to the inventory-based policy, lower initial inventory levels have been needed: \( S=[30 25 70 250 150] \).

With this policy, inventory levels are completely stabilized in a few periods but production fluctuations remain important, of the same order of magnitude as for the inventory-based policy.
Demand fluctuations are almost completely absorbed by the mean demand-driven policy. Results show that under the mean demand-driven policy, demand variations generate important production and inventory fluctuations along the supply chain. The mean demand-based policy considerably attenuates the final demand disturbances at all the stages of the supply chain, except for the final stage, for which important inventories are needed.

### Variance analysis for the three policies

The three policies have been simulated on series of 300 randomly generated demand trajectories for final products 1 and 2. The mean demand values have been kept constant \((d_1 = 20, \ d_2 = 15)\). Demand variances associated to the uniform distributions have been evaluated and indicated in Table 1. These variances are compared with production and inventory variances for the 5 products. In interpreting the results, it is important to take into account the bill of materials, which naturally amplifies for components the means and variances of production and inventory levels.

<table>
<thead>
<tr>
<th>Products</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand variance</td>
<td>134</td>
<td>75</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Inventory-based policy</td>
<td>Production variance</td>
<td>11</td>
<td>20</td>
<td>100</td>
<td>1500</td>
</tr>
<tr>
<td>Order-based policy</td>
<td>Production variance</td>
<td>135</td>
<td>75</td>
<td>850</td>
<td>10000</td>
</tr>
<tr>
<td>Demand-based policy</td>
<td>Production variance</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>Inventory variance</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td>Inventory variance</td>
<td>500</td>
<td>300</td>
<td>10</td>
<td>300</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 1 Variance comparison for the three policies

From this table, it is clear that under the inventory-based policy, demand variations generate important production and inventory fluctuations along the supply chain. The order-based policy dampens oscillations of inventory levels but not those of production levels. The mean demand-based policy considerably attenuates the final demand disturbances at all the stages of the supply chain, except for the final stage, for which important inventories are needed.

### Conclusion

Fluctuations of production and inventory levels are typical of multi-level material replenishment processes with significant delays and fluctuating demands. In supply chains, this phenomenon, known as the “bullwhip effect” has been observed in many practical cases. By simulation of analytical models, the study has shown that combination of classical local ordering policies tends to sustain or even amplify this effect. Two classical local policies have been studied: an inventory-based policy, inspired from the Kanban technique, and an order-based policy, inspired...
from MRP. In both case, combination of local feedback loops reinforces the oscillations and may generate instability. On the contrary, smoother production and inventory levels have been observed under the so-called mean demand-driven policy. Under this policy, the partner enterprises do not directly influence each other. They rather respond to integrated (or averaged) demand evolutions in parallel, global consistency in production and assembly being achieved through respect of the bills of materials.

REFERENCES


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is a Research Director of the French Research Center (CNRS). He graduated from the Ecole Nationale Supérieure de l'Aéronautique et de l'Espace, Toulouse, France, in 1974, received a MS degree in Industrial Engineering from Stanford University, California (1975), a Docteur-Ingénieur degree from INSA-Toulouse in 1978 and a degree of Docteur d'Etat from University Paul Sabatier, Toulouse in 1982. He joined the Laboratory for Analysis and Architecture of Systems of CNRS (LAAS-CNRS) in 1976, and since 1979, he has held a research position at CNRS. Since 2004, he has been on leave from LAAS-CNRS and has joined the LISA lab of Angers University. His research interests are in the fields of systems theory, optimization, control under constraints, discrete event decision systems and manufacturing systems. For more information on his work, please consult the web site http://www.istia.univ-angers.fr/~hennet or write to hennet@laas.fr.