ROBUST INVARIANT CONTROLLERS FOR CONSTRAINED LINEAR SYSTEMS

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Abstract

A state-feedback eigenstructure assignment method is described for providing a robust solution to some linear regulation problems under constraints on the state or on the input vector. The main result is that if the initial state vector belongs to a particular bounded set, the trajectories emanating from it will respect the constraints in a robust way, that is even if the system dynamics are slightly perturbed.

1. Introduction

Unlike bang-bang controllers, positively invariant regulators provide smooth solutions to many constrained control problems. They are based on the construction of a feasible domain in which the constrained vector (state, input or output) is compelled to belong and to converge to the set value (usually chosen to be the zero vector). This convergence is measured in terms of a norm (or a semi-norm) in the state-space, which is bound to decrease from any feasible initial point. Several design techniques based on this approach have been proposed for linear systems subject to constraints on the state vector and/or on the input vector [4], [8]. And recently, a structural interpretation of the positivity of a norm (or a semi-norm) in the state-space, which is the existence of a matrix $P$, $H_0$, $G$, $\omega$ and $\epsilon_n$. These conditions are necessary and sufficient for the invariance of the S.S.P. $S(P, \epsilon_n)$. with $x_k \in \mathbb{R}^n$ for any $k \in \mathbb{N}$, $A_0 \in \mathbb{R}^{n \times n}$. If this system is Lyapunov-stable, some domains of its state space have the following property of positive invariance.

Definition 1 : Positive Invariance

A nonempty set $\Omega$ is a positively invariant set of system (1) if and only if for any initial state $x_0 \in \Omega$, the complete trajectory of the state vector, $x_k$, remains in it. Or, equivalently, $\Omega$ has the property $A_0 \Omega \subseteq \Omega$.

To solve generic cases of symmetrical linear constraints, the analysis will be focused on the invariance conditions obtained when the set is a symmetrical polyhedron. A symmetrical polyhedron $S(G, \omega)$ can be defined by:

$$S(G, \omega) = \{ x \in \mathbb{R}^n : -\omega \leq Gx \leq \omega \}$$

with $G \in \mathbb{R}^{n \times n}$, $\omega$ a positive vector in $\mathbb{R}^n$. By convention, the inequalities between vectors are component-wise. From a classical result [2], [5], a necessary and sufficient condition for positive invariance of $S(G, \omega)$ is the existence of a matrix $H_0 \in \mathbb{R}^{n \times r}$ such that:

$$H_0 G = GA_0$$

$$|H_0| \omega \leq \omega$$

Definition 2 :

Simplicial Symmetrical Polyhedron (S.S.P.)

A symmetrical polyhedron $S(G, \omega)$ defined by (2) is said to be simplicial if and only if $G \in \mathbb{R}^{n \times n}$, $\text{rank}(G) = n$ and $\omega$ is a vector in $\mathbb{R}^n$ with strictly positive components.

Proposition 1 A necessary and sufficient condition for system (1) to admit a S.S.P. as a positively invariant set is the existence of a matrix $P \in \mathbb{R}^{n \times n}$ such that:

$$\|PA_0P^{-1}\|_{\infty} \leq 1$$

Proof

Sufficiency : Suppose the existence of a matrix $P \in \mathbb{R}^{n \times n}$ satisfying condition (5). Set $e_n = [1, \ldots, 1]^T$, $(\epsilon_n \in \mathbb{R}^n)$, and $H = PA_0$. We have
\[
\begin{cases}
HP = PA_0 \\
|H|e_n \leq \epsilon_n
\end{cases}
\]
These conditions are necessary and sufficient for the invariance of the S.S.P. $S(P, \epsilon_n)$.
Necessity: Suppose that system (1) admits as a positively invariant set the S.S.P. \( S(G, \omega) \). Then, from the basic property quoted above, there exists a matrix \( H_0 \in \mathbb{R}^{n \times n} \) satisfying relations (3), (4). Let \( G_i \) denote the ith row of matrix \( G \). Each \( \omega_i \) being strictly positive, each inequality \(-\omega_i \leq G_i x \leq \omega_i \) is equivalent to \(-1 \leq P_i x \leq 1 \) with \( P_i = 1/\omega_i G_i \).

Then, we can define \( P = [P_1^T, ..., P_n^T]^T \), \( H = \text{Diag}\{ \frac{1}{\omega_1} \} H_0 \text{Diag}\{ \omega_1 \} \). The set of constraints (2) defining the S.S.P. \( S(G, \omega) \), can be equivalently rewritten: \(-e_n \leq P x \leq e_n \). We obtain: \( H P = P A_0 \) and from the definition of \( H \) and some matrix properties, \( |H| = \text{Diag}\{ \frac{1}{\omega_1} \} |H_0| \text{Diag}\{ \omega_1 \} \). Thus, if we define \( \omega' = |H_0| \omega, 0 \leq \omega' \leq \omega \), we obtain
\[
|H e_n = \text{Diag}\{ \frac{1}{\omega_1} \} \omega' \leq e_n
\]
And, by definition of the infinite norm, relation (6) is equivalent to \( \|H\|_{\infty} \leq 1 \). □

Systems admitting positively invariant S.S.P.s can be analyzed in terms of stability, spectrum and robustness.

Stability: From a classical property of matrix norms, we have
\[
\rho(A_0) = \rho(P A_0 P^{-1}) \leq \|P A_0 P^{-1}\|_{\infty},
\]
where \( \rho(A_0) \) is the spectral radius of \( A_0 \). Therefore, condition (5) implies \( \rho(A_0) \leq 1 \), and the equality can only be obtained for simple eigenvalues. Then, it is clear that Lyapunov stability of system (1) is a necessary condition for the existence of a positively invariant S.S.P.. The converse property is obviously not true. As stated before, stability is sufficient for the existence of positively invariant domains, but it is not sufficient for the existence of positively invariant S.S.P.s.

A spectral sufficient condition: A sufficient condition of existence of an S.S.P. for system (1) can be obtained for a particular choice of a candidate matrix \( P \). Consider a generalized real Jordan form of \( A_0 \),
\[
\Lambda = Q A_0 Q^{-1}
\]
Each row of matrix \( Q \), denoted \( Q_i \), is a generalized real left eigenvector of \( A_0 \). If all the eigenvalues of \( A_0 \), denoted \( \mu_i + j\sigma_i \) satisfy the condition:
\[
|\mu_i| + |\sigma_i| < 1
\]
it can be shown [2] that \( (I_n - |\Lambda|) \) is an M-matrix. Then, from a classical property of M-matrices [1], there exists a strictly positive vector, \( \rho \) such that
\[
(I_n - |\Lambda|) \rho > 0
\]
If we select as the current row of matrix \( P \), \( P_i = 1/\rho_i Q_i \), we obtain, for \( H = \text{Diag}\{ \frac{1}{\rho_1} \} A \text{Diag}\{ \rho_1 \} \), the invariance conditions \( H P = P A_0 \) ; \( \|H e_n \leq e_n \). Therefore, spectral conditions (9) on the eigenvalues of \( A_0 \) are sufficient for the existence of a positively invariant S.S.P.

Robustness: Suppose now that the state matrix of the system, \( A_0 \) is either imperfectly known or subject to unpredicted bounded variations. What becomes of the invariance properties in such cases of structured uncertainties?

The perturbed state matrix is denoted \( A_0 + \Delta \). We want to evaluate the admissible domain of matrix \( \Delta \) for which the invariance property of the S.S.P. \( S(P, e_n) \) remains valid. Under condition (5), we can define \( \gamma \) with \( 0 \leq \gamma \leq 1 \) and:
\[
\|P A_0 P^{-1}\|_{\infty} = \gamma
\]
Invariance of \( S(P, e_n) \) will be maintained for the perturbed system if and only if:
\[
\|H \Delta\|_{\infty} \leq \|P \Delta P^{-1}\|_{\infty} + \gamma
\]
with by definition \( H \Delta = P(A_0 + \Delta) P^{-1} \). From Minkowski inequality, we have:
\[
\|H \Delta\|_{\infty} \leq \|P \Delta P^{-1}\|_{\infty} + \gamma
\]
And condition (12) holds if the following condition is satisfied:
\[
\|P \Delta P^{-1}\|_{\infty} \leq 1 - \gamma
\]
A stronger sufficient condition can then be derived:
\[
\|\Delta\|_{\infty} \leq \frac{1 - \gamma}{\|P\|_{\infty} \|P^{-1}\|_{\infty}}
\]
This bound measuring the robustness of the invariance property will be sufficiently large if:

- The norm \( \gamma \) is much smaller than 1
- The condition number of matrix \( P \): \( k_{\infty}(P) = \|P\|_{\infty} \|P^{-1}\|_{\infty} \) is small enough. The best possible value is 1.

Robustness of controlled invariance
Consider now the case of a controlled linear discrete-time system under a linear state feedback law:
\[
x_{k+1} = A_0 x_k + A + B F
\]
with \( x_k \in \mathbb{R}^n \), \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), \( m \leq n \), \( \text{rank}(B) = m \) and \( F \in \mathbb{R}^{m \times n} \).

Robust invariance of an S.S.P. The addressed problem is to find a gain matrix \( F \) such that a given S.S.P., \( S(P, e_n) \) should be a positively invariant set of the closed-loop system. The following Corollary can be obtained as a direct consequence of Proposition 1.

Corollary 1 A necessary and sufficient condition for the S.S.P. \( S(P, e_n) \) to be a positively invariant set of system (15) is the existence of a feedback gain matrix \( F \in \mathbb{R}^{m \times n} \) such that
\[
\|P(A + BF) P^{-1}\|_{\infty} \leq 1
\]
A possible design technique can be derived from relation (16). It is analogous to the algorithm presented in [8]. The matrix \( H = P(A + BF)P^{-1} \) is decomposed into:

- the matrix of its non-negative elements: \( H^+ \), with \( H^+_{ij} = \max(H_{ij}, 0) \)
- the matrix of its non-positive elements: \( H^- \), with \( H^-_{ij} = \max(-H_{ij}, 0) \)

so that \( H = H^+ - H^- \) and \( |H| = H^+ + H^- \). The technique consists in solving the following linear programming problem, with \( \gamma \) and the coefficients of matrices \( H^+, H^-, F \) as unknown variables:

\[
\begin{align*}
\text{Minimize} & \quad \gamma \\
\text{under} & \quad (H^+ + H^-)e_n - \gamma e_n \leq 0 \\
& \quad (H^+ - H^-)P - PB\gamma F \leq PA \quad (17)
\end{align*}
\]

If the optimal solution of this problem, denoted \( (\gamma^*, H^{+,*}, H^{-,*}, F^*) \), is such that \( \gamma^* < 1 \), the closed-loop system with state matrix \( A + BF^* \) admits \( S(P,e_n) \) as a positively invariant set. Note that in this section, \( P \in \mathbb{R}^{n \times n} \) is supposed to be given and full-rank. If the closed loop system is subject to the same type of additive perturbations on its state matrix and if the nominal value of \( A + BF^* \) verifies \( \gamma^* < 1 \), the robustness of the invariance property is also maximized by the choice of \( F^* \) as the gain matrix. The optimal robustness property of this design technique derives from the fact that the optimal value of the criterion, \( \gamma^* \), maximizes the right-hand term of relation (13).

**Non robust invariance of unbounded polyhedra:** In many cases, such as linear constraints on the input or on the output vector, the domain of constraints, represented in the state space, is unbounded. Here, we consider the following symmetrical case: \( x_0 \in S(G, e_v) \), with \( G \in \mathbb{R}^{r \times n} \), \( r \leq n \), \( \text{rank}(G) = r \), and

\[
S(G, e_v) = \{ x \in \mathbb{R}^n : -e_v \leq Gx \leq e_v \} \quad (18)
\]

The linear constrained regulation problem can be solved by imposing the closed-loop positive invariance of the largest set \( \Omega \) such that \( \Omega \subset S(G, e_v) \). Some extra stability condition are then also needed [8].

As developed in [6], a necessary condition for the invariance of \( S(G, e_v) \) is that the subspace \( \text{Ker} G \) should be \((A, B)\)-invariant in the sense of Wonham [9]. Suppose that this property is satisfied with the state feedback gain matrix \( F \). Suppose now that the closed-loop state matrix, \( A_0 \), is replaced by \( A_0 + \Delta \), with matrix \( \Delta \) unknown and possibly full-rank. Then, it can be easily shown that the invariance property of \( \text{Ker} G \) cannot be maintained under any choice of \( F \).

To obtain a robust invariant control scheme, it is possible to construct a bounded positively invariant set, \( \Omega \), included in \( S(G, e_v) \). To satisfy constraints (2) all along the trajectory, the admissible initial states of the system will then be restricted to \( \Lambda \). Clearly, a simple way to build such a candidate set \( \Lambda \) is to complete matrix \( G \) by \( n-r \) independent row vectors, constituting a matrix \( G' \), to make up a non-singular matrix in \( \mathbb{R}^{n \times n} \), \( P = \begin{bmatrix} G' \\ G \end{bmatrix} \).

The design problem can then be reduced to finding a gain matrix \( F \) for which the S.S.P. \( S(P, e_v) \) is positively invariant in a robust way. The choice of matrix \( G' \) and of the "nominal" eigenstructure of the closed-loop system should be made so as to maximize the robustness of the invariance property. Once matrix \( G' \) has been selected, the program (17) gives the optimally robust invariant solution, if \( \gamma^* < 1 \). But the problem of optimally choosing \( G' \) cannot be directly solved. An alternative technique for solving this problem in a robust way will be presented in section 3. It is not based on the linear program (17) but on eigenstructure assignment.

**3. A robust design technique**

The technique which will now be presented applies when \( \text{rank}(G) \leq m \). It is based on eigenstructure assignment, with the candidate matrix \( H_0 \) of relations (3) and (4) under the real Jordan form.

**The basic method for unperturbed systems**

At the structural level, \( y_k = Gx_k \) can be interpreted as an output vector. However, the considered controls are linear state (and not output) feedback laws.

Let \( P(\lambda) = \begin{bmatrix} \lambda I - A & -B \\ G & 0_{s \times m} \end{bmatrix} \) be the system matrix.

In the case of systems with perfectly known parameters, it may be desirable to construct a set of admissible initial states which is unbounded in some directions. To generate positively invariant domains with infinite directions, it suffices to select a set of \( s \) stable eigenvalues, \( \lambda_i \), with \( s \leq n - r \), with their associated directions in the state and input spaces, \((v_i, w_i)\), satisfying the zero equation (19), with vectors \( v_i \) independent:

\[
P(\lambda_i) \begin{bmatrix} v_i \\ w_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (19)
\]

Denote \( S \) the subspace spanned by \( V_1 = (v_1, \ldots, v_s) \). \( S \subset \text{Ker} G \) since \( GV_1 = 0 \). The other selected closed-loop poles \( \lambda_i = \mu_i + j\sigma_i \) satisfy inequality (9). Their associated generalized real eigenvectors \( v'_i \) span a subspace \( R \subset \mathbb{R}^n \) such that \( R \oplus S = \mathbb{R}^n \).

They are the column-vectors of a matrix \( V_2 \) which satisfies: \( GV_2 = D \), for some selected matrix \( D = \begin{bmatrix} d_{11}, \ldots, d_{n-s} \end{bmatrix} \), with \( d_l \neq 0 \) \( \forall l \in \{1, \ldots, n-s\} \) (3). For each selected eigenvalue, \( \lambda_i \), and each selected projected component \( d_l \), the associated directions \( v_i \) and \( w_i \) satisfy:

\[
P(\lambda_i) \begin{bmatrix} v_i \\ w_i \end{bmatrix} = \begin{bmatrix} 0 \\ d_l \end{bmatrix} \quad (20)
\]
Finally, let $V = [V_1 | V_2]$ be the matrix of the desired real generalized eigenvectors, and $W = [W_1 | W_2]$ the associated input directions. The feedback gain matrix providing the desired eigenstructure assignment is $F = W V^{-1}$.

In the sequel, we assume, for simplicity, that $r \leq m$ and that system $(A, B, G)$ does not have any unstable invariant zero. In this case, it is possible to construct $V_1$ and $V_2$ such that $s = n - r, S = \text{Ker } G$, and $D = I_r$.

**Robustness improvement**

The choice of the desired "nominal" closed-loop poles is basic for robustness issues. In the robust version of the placement algorithms, the invariant zeros of the system can be selected as closed-loop eigenvalues only if they satisfy the spectral condition (9). Then [7], the aim of a robust eigenvector assignment design is to construct a basis of eigenvectors as "close" as possible to orthogonality. The major difference between the various measures of robustness differ on the way to evaluate this "distance" to orthogonality. It is worth recalling that a decrease in the conditioning of the matrix of eigenvectors lowers the sensitivity of the eigenvalues to structured perturbations. In section 2, we have defined the condition number of matrix $P$ as $k_{\infty}(P) = \|P\|_{\infty} \|P^{-1}\|_{\infty}$. A more classical definition of the condition number is obtained when using the square norm: $k_2(P) = \|P\|_2 \|P^{-1}\|_2$. Here, by construction, $P = \begin{bmatrix} G' & G \\ V_1 & V_2 \end{bmatrix}$. The matrix of associated eigenvectors is $V = [V_1 | V_2]$. It satisfies:

$$PV = \begin{bmatrix} G' \\ G \end{bmatrix} [V_1 | V_2] = \begin{bmatrix} I_{n-r} & 0_{n-r-r} \\ 0_{r+n-r} & I_r \end{bmatrix}$$  \hspace{1cm} (21)

For a selected set of admissible eigenvalues, the maximal infinite norm of an admissible perturbation matrix $\Delta$ will be obtained at the optimum of the following problem:

$$\text{Minimize} \quad k_{\infty}(P) = \|P\|_{\infty} \|P^{-1}\|_{\infty} \quad \text{under} \quad PV = I_n, \quad G \text{ given}$$  \hspace{1cm} (22)

For each selected eigenvalue $\lambda_i$, compute the kernel of $P(\lambda_i), \quad K_i = \begin{bmatrix} N_i \\ M_i \end{bmatrix}$. Each candidate eigenvector of $\text{Ker } G$, $v_i$, associated to the eigenvalue $\lambda_i$ for $i \in (1, \ldots, n - r)$ is written: $v_i = N_i k_i$. If we impose $\|v_i\| = 1$, there is some choice for $k_i$ only if $\text{rank}(N_i) > 1$. Each candidate eigenvector $v_i$ for $l \in (n - r + 1, \ldots, n)$ of a complementary subspace of $\text{Ker } G$, associated to the eigenvalue $\lambda_i$ is written:

$$v_l = v_l^0 + N_l k_l$$  \hspace{1cm} (23)

where $v_l^0$ is a particular solution of equation (20). There is some degree of freedom in the choice of $k_l$ in equation (23) if $\text{rank}(N_l) \geq 1$. Matrix $V$ can be written:

$$V = [0_{n*(n-r)} | V_0] + \text{Diag}\{N_1, \ldots, N_n\} \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}$$  \hspace{1cm} (24)

with $V_0 = [v_{n-r+1}^0, \ldots, v_n^0]$.

Problem (22) is difficult to solve. It can be simplified by replacing the criterion $k_{\infty}(P)$ by $k_2(P)$. An iterative algorithm such as method 0 in [7] can then be used for decreasing the value of $k_2(P)$. Once a possible matrix of eigenvectors, $V$ has been computed, it can be iteratively improved if there are some degrees of freedom in the choice of vectors $k_i$ or $k_l$. Such an improvement can be obtained by:

- removing one of the eigenvectors, say $v_l$, from the basis,
- computing the vector $\tilde{y}$ orthogonal to the uncompleted basis of eigenvectors,
- replacing the missing eigenvector by the projection of $\tilde{y}$ on the $l$th transmission subspace.

**An example**

The robust invariant design technique was used on the example defined by the following data:

$$A = \begin{bmatrix} 0 & 1.87 & 0 & 0 \\ -0.47 & 1.92 & -0.80 & 0 \\ 0 & 0.52 & 1.45 & 0 \\ 1.36 & -1.66 & 0.67 & 1.34 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} -0.69 & 0 & 0 \\ -0.59 & 0 & 0.91 \\ 0 & 0 & -0.76 \\ -0.42 & 0 & 0.26 \end{bmatrix}$$

The open-loop system has eigenvalues: $(1.34, 1.45 \pm 0.78i, 0.47)$. The state constraints are defined by $G = \begin{bmatrix} 22.48 & -68.38 & 1.29 & -0.11 \\ -18.69 & 63.31 & 1.33 & 0.58 \end{bmatrix}$. The selected closed-loop eigenvalues associated to $\text{Ker } G$ are: $\lambda_1 = 0.9, \lambda_2 = 0.75$. The other closed-loop poles have been chosen as follows: $\lambda_3 = 0.69, \lambda_4 = 0.8$. We then get:

$$F = \begin{bmatrix} 3.44 & -5.79 & 2.56 & 1.22 \\ 0.80 & -1.58 & 0.86 & 0.02 \end{bmatrix}$$

The optimized conditioning is $k_2(P) \approx 44.8$. A sharper sensitivity index is:

$$c_{\text{max}} = \max_{j=1, \ldots, n} \frac{||P_j|| ||V_j||}{||P_j V_j||}$$

Here, we obtain $c_{\text{max}} \approx 20.3$ with $P = \begin{bmatrix} G' \\ G \end{bmatrix}$ and $G' = \begin{bmatrix} 58.8 & 19.7 & -45.4 & 21.9 \\ 44.2 & 38.0 & -34.0 & -14.8 \end{bmatrix}$.

An admissible perturbation matrix satisfying (13) is now generated. The domain of constraints $S(G, e_2)$ is positively invariant for the nominal and for the perturbed closed-loop matrix from the initial points in
There is a matrix $S(P, e_n)$. In particular, we see on Fig. 1 the admissible trajectories for the nominal and for the perturbed system, from $x_{01} = [0.32 \ 0.10 \ 0.48 \ 0.07]^T$, which satisfies $Px_{01} \simeq e_4$. But we can also observe on Fig. 1 that the two other trajectories violate the constraints in $G$. This is due to the fact that their initial points $x_{02} = [0.88 \ 0.28 \ 0.06 \ -0.19]^T$ and $x_{03} = [1.44 \ 0.45 \ -0.36 \ -0.46]^T$, satisfy the constraints in $G$ but not in $G'$.

Figure 1: Projected trajectories from $x_{01}$, $x_{02}$ and $x_{03}$ for the nominal system (- -) and from $x_{01}$ (- -), $x_{02}$ (--) and $x_{03}$ (--) for the perturbed system.

**The case of constrained inputs**

Suppose that at any time $k$ the control vector $u_k$ should belong to the symmetrical polytope (in the input space) $\Sigma(T, e_m)$ defined by (25), with $T$ a full-rank matrix in $\mathbb{R}^{m+n}$:

$$\Sigma(T, e_m) = \{ u \in \mathbb{R}^m ; \ -e_m \leq Tu \leq e_m \} \quad (25)$$

Under a linear state feedback control law, $u_k = Fx_k$, constraints on the input vector define an unbounded polyhedron, $S(TF, e_m)$ in the state space. But this polyhedron is not given. It is constructed from the gain matrix to be computed, $F$. Yet, the eigenstructure assignment scheme presented in chapter 3.1 can also apply to this case, after some minor changes [6]:

- $n - m$ stable poles of $A$ should be used as closed-loop poles to obtain the closed-loop invariance of the subspace $\ker TF$.
- The $m$ other poles, $\lambda_l$ for $l = (n-m+1, \ldots, n)$, should satisfy the spectral conditions (9).

Their associated eigenvectors can be computed by solving the sets of equations:

$$\begin{bmatrix} \lambda I - A & -B \\ 0_{n \times n} & T \end{bmatrix} \begin{bmatrix} e_1 \\ w_l \end{bmatrix} = \begin{bmatrix} 0 \\ d_l \end{bmatrix},$$

with $d_l = [0 \ldots 0]_l^T$. To obtain a robust control scheme, it then suffices to restrict the initial state to the S.S.P. $S(P, e_n)$, with $P = [V_1 \ | \ V_2]^{-1}$, if the open-loop poles kept as closed-loop poles also satisfy the spectral condition (9).

**4. Conclusion**

In the framework of linear systems with constant state-feedback, a robust solution to control problems under symmetrical linear constraints can be obtained if it is possible to make positively invariant some bounded symmetrical polyhedron of the state space. Such a polyhedron can be constructed partly from the constraints of the problem and from other constraints selected so as to maximize the robustness of the control scheme. A simple algorithm, based on eigenstructure assignment, provides a solution to the constrained control problem for the nominal (unperturbed) system. This solution can be used as an initial solution for the robust constrained control problem. The method presented in this paper can easily be translated to the case of input constraints and also to continuous-time systems ([3]).

**References**


