A Cooperative Game Based Approach for Resource Pooling and Profit Sharing in Supply Chains

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Abstract: This study addresses the strategic problem of supply chain formation on the basis of the quantities and prices of end-products to be manufactured and sold on a market. The manufacturing process is planned from the products bills of materials (BOM) and distributed on the resources available in the enterprises network. Resources are modeled as capacitated systems with piecewise-linear throughput functions of the workload. The problem of maximal profit generation and sharing among the firms of the network is analyzed and solved as a cooperative game. The proposed profit sharing rule is constructed from the dual of the profit maximization problem. It is both efficient and rational, with more fairness than the Owen rule of classical Linear Production Games. Copyright © 2012 IFAC

Keywords: Supply networks, Game theory, Manufacturing systems, Resource clearing functions, Linear production games, Duality.

1. INTRODUCTION

Based on the cooperative game theory approach, a supply chain can be modelled as a coalition of partners pooling their resources and sharing the same utility function (profit). The works of Cachon and Netessine (2004) and Nagarajan and Sošić (2008) provide convincing interpretations of supply chain design problems as cooperative games.

In order to form a supply chain, the enterprises should select the quantities of end-products that they plan to sell on the market and use their means of production in agreement with the product structure and the operation sequence of the goods to be manufactured. Such an arrangement should be performed in the most profitable manner for all the enterprises involved. The choice of the most efficient coalition of enterprises sharing their manufacturing and logistic resources is the main issue of the Linear Supply Chain Game (LSCG) studied in (Hennet and Mahjoub, 2010). As a complementary issue for the LSCG, the benefits created by cooperatively organizing production should be rationally and fairly rewarded among the members of the coalition, so as to stabilize the involvement of the supply chain members.

Along this research line, the LSCG has been formulated as an extension of the linear production game (LPG) studied by Shapley and Shubik (1972) and Owen (1975). One limitation of this formulation is the difficulty to combine rationality and fairness in the allocation policy, due to the fact that shadow prices of resources drop to zero when their capacity is in excess in the coalition. This study considers manufacturing resources not only from the capacity viewpoint but also from their utilization conditions, by introducing in the model the cost of the WIP (Work in Progress). Then, the PLSCG (Piecewise Linear Supply Chain Game) defined and studied in this paper represents the saturation constraints on resources in a more detailed manner, while preserving linearity of the model. The main reason for maintaining the linearity property is to remain in the scope of the LPG, with the resulting possibility of using the Owen set profit allocation rule (Owen 1975) as a rational profit allocation policy.

The main originality of the PLSCG formulation introduced in this paper is that it represents the saturation constraints on resources in a more precise and operational manner, by taking into account the influence of the workload on the throughput. Then, the function of the chain integrates as a positive term the anticipated revenue to be obtained from the sale of the end-products on the market and as negative terms manufacturing costs and holding costs of all the products and components.

The superadditivity property of the PLSCG demonstrated in the paper allows for an easy computation by Linear Programming of the maximal profit achievable from the formation of a supply chain in the network of enterprises. Then, a stable profit allocation rule is constructed as a generalization to the PLSCG of the Owen set concept used for LPG (Owen 1975, Van Gellekom et al. 2000). A practical advantage of the proposed PLSCG allocation policy over the classical Owen policy for the LSCG is that it improves the fairness of the profit allocation rule by integrating the costs of resource utilization before reaching the capacity limits.

Section 2 introduces the supply chain formation problem declined from the product structure of the goods to be sold on the market. Section 3 solves the PLSCG by optimizing the expected profit of the supply chain and constructing a stable profit allocation policy. An illustrative numerical example is provided in Section 4, and Section 5 concludes the paper.
2. A PRODUCT STRUCTURE DRIVEN SUPPLY CHAIN FORMATION PROCESS

2.1 The BOM-related multi-stage model

The concept of a supply chain concentrates some major features of business organization in today’s Society. It characterizes a network of autonomous production units connected through an information and communication network and through a logistic network. In the recent literature, such a network is sometimes called “cloud of collaborative enterprises” or “cloud supply chain” (Lindner et al., 2010). Typically, the information network carries commercial proposals, products orders, manufacturing and delivery protocols. The logistic network used to transport goods and products may be owned by enterprises in the network or by subcontractors. In any case, a supply chain can be viewed as a multistage production and transportation system in which the different transformation stages are performed by different enterprises. Requirements planning models (Baker 1993) can then be used to define and distribute responsibilities and manufacturing orders among the partners. In this view, the product structure supports the enterprise network organization, especially under an extended view of the BOM (Bill of Materials), such as the G-BOM (Generic BOM, (Lamothe et al., 2005)), integrating product families rather than simple products.

A convenient graphical tool to represent multi-product multi-stage bills of materials is the gozinto graph proposed by Vazsonyi (1955). An example of such a graph is represented in Fig. 1. According to the classical decomposition of products structures into levels (see e.g. Salomon, 1991), the three end-products numbered 1, 2, 3 are level 0 products, the two products numbered 4, 5 are level 1 products and the three products 6, 7, 8 are level 2 products.

![Fig. 1 An eight-product structure](image)

More generally, level 0 products constitute the set of end-products (or families of products) \( i \in \{1, \ldots, g\} \). Intermediate and primary products are numbered in the increasing order of their level. The level of product \( i \), for \( i=g+1, \ldots, n \) is the maximal number of stages to transform product \( i \) into a final product. Each production stage is supposed to have several input products but only one output product. The BOM technical matrix \( \Pi \), is defined as follows: according to a given manufacturing recipe, production of one unit of product \( i \) requires the combination of components \( l \in \{1, \ldots, n\} \) in quantities \( \pi_{il} \). It can be noted that under a level-consistent ordering of products, matrix \( \Pi \) has a simple lower triangular structure (Hennet 2003). For example, matrix \( \Pi \) associated with the structure of Fig. 1 is written as follows:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Additionally, multistage production by several producers highly differs from multistage production by a single producer because of the need for negotiation, contracts and higher coordination requirements. It also carries new possibilities in the design stage for selecting partners, sharing resources, risks and rewards.

Let \( \mathcal{N} \) be the set of \( N \) enterprises who candidate to be part of the supply chain to be created. Each candidate enterprise is characterized by its production resources: manufacturing plants, machines, work teams, robots, pallets, storage areas.

The supply chain model is prospectively formulated over a reference time horizon and in stationary conditions. Thus, lead times are not included in the model and material supply is supposed perfectly coordinated with manufacturing processes. Then, let \( \mathbf{X} = ((x_{ij})) \) be the matrix of the quantities of product \( i \) produced (or obtained by exchange) at firm \( j \) and \( \mathbf{y} = (y_1, \ldots, y_n)^T \) be the output vector during the reference period. The components of this matrix and vector are the variables of the design problem. For simplicity, quantities per period (or throughputs) are supposed continuous: \( x \in \mathbb{R}_{n \times n}, y \in \mathbb{R}_n \). The problem can be formulated in terms of the global throughput vector, denoted \( \omega = [\omega_1 \cdots \omega_N]^T \) and related to matrix \( \mathbf{X} \) through the elementary summation relations (1):

\[
\omega_i = \sum_{j=1}^N x_{ij} \quad \text{for} \quad i=1, \ldots, n. \tag{1}
\]

Equations (1) for \( i=1, \ldots, n \) are summarized in vector form:

\[
\omega = \mathbf{X} \mathbf{1}_N, \quad \mathbf{1}_N \text{ being the unit vector of dimension } N.
\]

The output vector can be computed from the global throughput vector by the following relation:

\[
y = (I - \Pi)\omega, \quad \text{with } I \text{ the } n \times n \text{ identity matrix}. \tag{2}
\]

From the structure of matrix \( \Pi \), matrix \( (I - \Pi) \) is regular and matrix \( (I - \Pi)^{-1} \) is lower triangular (with 1s on the diagonal) and nonnegative. Then, for a nonnegative output vector \( y \), the
global throughput vector is also nonnegative since it is expressed as follows:

\[ \omega = (I - \Pi)^{-1} y. \]  

(3)

2.2 Resource capacity and WIP

Consider the \( R \) types of resources available in the network \( r=1, \ldots, R \). The amount of resource \( r \) available for enterprise \( j \) is denoted \( k_{rj} \), and the resource capacity matrix is defined as:

\[ K = ((k_{rj})) \in \mathbb{R}^{R \times N}. \]

A subset \( S \) of enterprises, with \( S \subseteq N \), can be represented by its characteristic vector \( e_S \in \{0,1\}^N \) such that:

\[
\begin{cases}
(e_S)_j = 1 & \text{if } j \in S \\
(e_S)_j = 0 & \text{if } j \notin S
\end{cases}
\]  

(4)

Let \( m_{ri} \) be the amount of resource \( r \) necessary to produce \( l \) unit of product \( i \). \( M = ((m_{ri})) \in \mathbb{R}^{R \times n} \).

The first issue addressed in this paper is how to represent resource capacity in a consistent manner with the resource saturation phenomena observed in practice. Basically, a resource such as a machine or transportation equipment is characterized by its decreasing efficiency relative to the load.

Classical capacity constraints used in aggregate production planning problems are simple saturation functions of the “all or nothing” type. Such capacity constraints are valid and will be used in our model. In particular, the capacity constraints restricted to a coalition \( S \subseteq N \) are written:

\[ M (I - \Pi)^{-1} y \leq Ke_S \]  

(5)

But constraints (5) are not sufficient to describe the saturation phenomena, because they do not represent the average workload in the system. Using the “Little law” (Little, 1961), systems with saturation functions completely described by (5) are associated with constant lead times. However, as stressed in Karmarkar (1993), lead times data show a superlinear increase of lead times with capacity utilization, and this property is true, at various magnitude levels, for any type of physical resources. In queuing theory, a lead time is classically decomposed into a processing time of constant expected value and a waiting time whose mean value increases with the mean population (or WIP, Work In Progress). Such convex lead time variations with the WIP are associated with concave output functions of the WIP called clearing functions, as represented on Fig. 2.

For resource \( r \) located at enterprise \( j \), the WIP is denoted \( w_{rj} \) and the throughput is denoted \( \theta_{rj} \). It is given by:

\[ \theta_{rj} = \sum_{i=1}^{n} m_{rij} x_{ij}, \]  

(6)

or, in matrix form: \( \Theta = MX \)  

(7)

In the subset of enterprises \( S \subseteq N \), constraints related to the type \( r \) resource owned by the enterprises in \( S \) take the following aggregated form:

\[ \theta_{rS} = \sum_{j=1}^{N} (e_S)_j \theta_{rj} \leq \sum_{j=1}^{N} k_{rj} \cdot (e_S)_j \]  

(8)

\[ w_{rS} \geq \alpha^1_r \theta_{rS} \]  

(9)

\[ w_{rS} \geq \alpha^2_r (\theta_{rS} - k^0_r) \]  

(10)

Constraint (8) is the standard capacity constraint included in the set of constraints (5) and related to throughput \( \theta_{rS} \) computed from (6). Constraints (8) and (9) express that the aggregated WIP of resource \( r \) is above the inverse clearing function of Fig. 3. Parameters \( \alpha^1_r \) and \( \alpha^2_r \) characterize resource \( r \), independently from the enterprise \( j \) owning it. They are both positive and satisfy \( \alpha^1_r < \alpha^2_r \). Similarly, parameters \( k^0_r \) satisfy \( 0 \leq k^0_r < k_{rj} \).

Equality in constraint (9) or (10) will be obtained through constraint saturation when minimizing \( w_{rS} \) in the criterion, as it will be the case by minimizing the holding costs.
Using model (8)-(10), for all the resources, we define two nonnegative vectors of resource dependent parameters: \([a_1^r \ldots a_R^r]^T, [a_1^s \ldots a_R^s]^T\), the diagonal matrices of dimension \(R \times R\) \(A_r\) and \(A_s\) such that \((A_1^r)_{rr} = a_1^r\), \((A_2^s)_{rr} = a_r^r\) and \(K_0 = ((k_j^0)^T)\).

3. THE PIECEWISE LINEAR SUPPLY CHAIN GAME

3.1 The profit maximization problem

Unit purchasing costs of primary products (products 6, 7, 8 in the example of Fig.1) and manufacturing costs of intermediate and end-products (products 1-5 in the example of Fig.1) are supposed fixed and given. They are noted \(c_i\) for \(i \in \{1, \ldots, n_i\}\) and in vector form \(c = [c_1, \ldots, c_n]^T\). The unit holding cost of workload \(w_r\) is denoted \(h_r\). In vector form, \(h = [h_1 \ldots h_R]^T\), \(w = [w_1 \ldots w_R]^T\).

End-products are the goods sold on the market at fixed and given market prices: \(p_i\) for \(i \in \{1, \ldots, n_i\}\), and \(p = [p_1, \ldots, p_n]^T\) with by convention \(p_i = 0\) for \(i \in \{g+1, \ldots, n\}\).

The profit expected from manufacturing and sale on the market of the vector of outputs \(y\) is given by: \((p^T y - c^T \omega)\), which can be re-written: \((p^T - c^T (I - \Pi)^{-1}) y\). Additional storage costs will be subtracted from this expression to formulate the total expected profit of the chain.

The profit maximization problem related to the possible supply chain formed by the enterprises in \(S \subseteq N\) can then be stated as follows.

Maximize \(v = \sum_{i \in N} p_i - c_i^T (I - \Pi)^{-1} y - h_i^T w\)

subject to

\[ M(I - \Pi)^{-1} y \leq K e_S \]
\[ A_r M(I - \Pi)^{-1} y - w \leq 0 \]  \(\text{(PS)}\)
\[ A_s M(I - \Pi)^{-1} y - w \leq A_S K e_S \]
\[ y \in \mathbb{R}_+^n, w \in \mathbb{R}_+^R, e_S \in [0,1]^N \]

In the cooperative game theory framework, each subset of enterprises \(S \subseteq N\) is considered as a possible coalition of players, with value function \(v(S)\) computed as the optimal criterion of the profit maximization problem \((PS)\). The set of problems \((PS)\) over all the possible coalitions \(S \subseteq N\) defines a cooperative game, called the Piecewise Linear Supply Chain Game, or LPSCG.

It can be noticed that any coalition \(S'\) such that \(S \subseteq S'\) satisfies \(e_S \leq e_{S'}\). Therefore, the optimal solution of \((PS)\) is feasible for \((PS)\) and, as a consequence, \(\psi(S') \geq \psi(S)\). The game \((N, \psi)\) is said to be superadditive (see e.g. Osborne and Rubinstein, 1994) and the following property derives from superadditivity.

**Property 1**

The maximal profit \(v^*\) that can be obtained by any coalition \(S \subseteq N\) is also obtained by the grand coalition \(N: v^* = \psi(N)\).

According to property 1, the maximum achievable profit with respect to any coalition \(S \subseteq N\) can be directly obtained by solving problem \((PS)\) for which \(e_N = 1_N\). This property is important for two reasons. The first one is that it allows for an easy computation of the maximal possible profit value by Linear Programming. The second one is that when vector \(e_S\) is given, problem \((PS)\) only contains continuous variable and the optimal value of its criterion is also the optimal value of the criterion of its dual problem, without any “duality gap”. This property will be used to compute a profit allocation policy for the enterprises in the network.

3.2 A profit sharing mechanism for the member enterprises

The key problem in forming a supply chain is to guarantee that it will be stable in the sense that the member enterprises will not want to separate from each other to create a more profitable chain. In the cooperative game theory, the set of stable profit allocations is called the “core” of the game. A profit allocation is noted \((u_i)_{i \in \{1, \ldots, n\}}\). It belongs to the core of the game \((N, \psi)\) if and only if it satisfies the following properties:

1. Efficiency:
   \[ \sum_{i \in N} u_i = v^* \]  \(\text{(11)}\)
2. Rationality:
   \[ \sum_{i \in S} u_i \geq v(S) \quad \forall S \subseteq N \]  \(\text{(12)}\)

Condition (12) indicates that for any player \(i \in N\), there is no coalition alternative in which he could obtain a strictly greater reward.

If holding costs were neglected, problem \((PS)\) could be reformulated, as in (Henmet and Mahjoub, 2010) to represent the game as a classical Linear Production Game (LPG):

Maximize \(v = g^T y\)

subject to \(A y \leq Ke_S\)  \(\text{(PS')}\)
\[ y \in \mathbb{R}_+^n, e_S \in [0,1]^N \]

with \(g = p^T - c^T (I - \Pi)^{-1}\), \(A = M(I - \Pi)^{-1}\).

Linear Production Games have been introduced by Shapley and Shubik (1972) and also studied by Owen (1975), Van Gellekom et al. (2000). The solution rule proposed by (Owen 1975) and called the Owen set, is constructed from the
optimal solution of the dual of problem \((P_N')\). The optimal dual variables are interpreted as the marginal costs (or shadow prices) of resources. In an Owen assignment, the payoff of each player equals the value of his resource bundle under the unit marginal cost of resources. Moreover, it has been shown in (Owen, 1975) that this vector of payoffs forms a subset of the core in this production game.

A similar construction can be achieved with problem \((P_N')\), with a more detailed evaluation of the cost of resources resulting from the introduction of the WIP in the model.

Three types of dual variables characterize the dual \((D_N)\) of problem \((P_N)\): variables \(z_r\), associated with the first set of constraints in \((P_N)\), variables \(\eta_1^r\) and \(\eta_2^r\) associated with the second and third sets of constraints in \((P_N)\). To obtain a more compact formulation of the dual problem, the following vectors of variables are introduced:

\[
z = (z_1, \ldots, z_R)^T, \quad \eta_1 = (\eta_1^1, \ldots, \eta_1^R)^T, \quad \eta_2 = (\eta_2^1, \ldots, \eta_2^R)^T.
\]

Problem \((D_N)\) is stated as follows.

Minimize \(\phi = q^T z + K_0 A_2 \eta_2\)

subject to

\[
(1 - \Pi)^T M^T (z + A_1 \eta_1 + A_2 \eta_2) \geq p - (1 - \Pi)^T c
\]

\[n_1 + \eta_2 \leq h\]

\[z \in \mathbb{R}_+^R, \quad \eta_1 \in \mathbb{R}_+^R, \quad \eta_2 \in \mathbb{R}_+^R.
\]

The coefficient of variable \(z_r\) in the objective function corresponds to the quantity of resource \(r\) available for production if the network:

\[
q = (q_1, \ldots, q_R)^T \text{ with } q_r = \sum_{j=1}^N k_{rj}.
\]

By analogy to the Owen set for LPG, a profit allocation policy \(u_i\) defining the profit allocation vector 

\[
u = [u_1, \ldots, u_N]^T\]

can be constructed from the optimal solution \((z^*, \eta_1^*, \eta_2^*)\) of problem \((D_N)\), in the form:

\[
u_j = \sum_{r=1}^R (k_{rj} z^*_r + \alpha_r^2 k_{rj} \eta_2^2), \quad \text{for } j = 1, \ldots, N
\]

By construction, \(\sum_{j=1}^N \nu_j = \phi^*\) and from the strong duality property, \(v^* = \phi^*\). The proposed allocation policy is efficient in the sense of Definition (11).

Rationality of this policy can also be shown, as for the Owen set, from the property that the constraints defining the dual problem of \((P_S)\) with \(e_S\) fixed, are the same for any coalition \(S\) (Van Gellekom et al. 2000).

Then, from the definition of the core, the following property is derived

**Property 2**

The feasible payoff profile \((u_i^*)_{i \in \mathbb{N}}\) defined by relations (14) belongs to the core of the Supply Chain Game (PLSCG).

The proposed profit allocation mechanism for the PLSCG has the same property of coalitional stability than the Owen set for the LPG. However, it has been observed that the Owen set solution of the LPG has the drawback of being unfair by not rewarding some enterprises having a positive marginal contribution to the global profit (Hennet and Mahjoub, 2010).

Typically, this unfairness mainly arises from the fact that in the optimal dual solution, resources in excess have null shadow prices: \(z_r^* = 0\) if resource \(r\) is in excess in the optimal production plan. Introduction of the piecewise linear resource saturation mechanism in the model has generated an additional term \(\eta_2^r\) related to the WIP of resource \((r,j)\) and this term becomes strictly positive as soon as throughput \(\theta_r\) satisfies \(\theta_r > K_0^r\). These properties are straightforward consequences of duality in Linear Programming. They will be illustrated on a numerical example.

4. A NUMERICAL EXAMPLE

Consider the BOM of the example presented in section 2.1, with three final products \((1, 2, 3)\), two intermediate products \((4, 5)\), and three primary products \((6, 7, 8)\). The vector of unit market prices is \(p = [100, 125, 130, 0, 0, 0, 0, 0]^T\).

Manufacturing costs are \(c = [5, 4, 4, 2, 3, 4, 5, 6]^T\). Five resources are necessary for the five products at the different manufacturing stages, with the following resource requirement matrix \(M\):

\[
M = \begin{bmatrix}
1 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 1 & 0 & 1 \\
2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 2 & 2 & 2 & 2
\end{bmatrix}
\]

Four enterprises are candidate for partnership in the supply chain. The amounts of the five resources owned by the 4 enterprises are represented in the following matrix:

\[
K = \begin{bmatrix}
0 & 100 & 100 & 0 \\
100 & 0 & 0 & 100 \\
0 & 20 & 20 & 20 \\
200 & 0 & 100 & 0 \\
200 & 100 & 0 & 100
\end{bmatrix}
\]

WIP holding costs are supposed only resource dependent and given by the vector: \(h = [0.5, 0.5, 0.5]^T\). Saturation parameters are supposed homogeneous for all the resources and all the enterprises, with \(\alpha_r^1 = 0.5,\)
$a^2_r = 2, \, k_{ij}^0 = k_{ij}^0 / 2$. The optimal total profit is obtained from the solution of the LP $(P_N)$:

$$v^* = 1115.7,$$

$$y_1^* = 28.57, \quad y_2^* = y_3^* = 0,$$

$$w^* = [85.71 \quad 200 \quad 54.29 \quad 157.14 \quad 400]^T.$$ 

The optimal solution of the dual $(D_N)$ is $\phi^* = 1115.7$. It is obtained for $z^* = [0 \quad 0.62 \quad 0 \quad 0.40]^T$, $\eta_1^* = [0 \quad 0 \quad 0 \quad 0 \quad 0]^T$ and $\eta_2^* = [0.5 \quad 1 \quad 0.5 \quad 1 \quad 0.5]^T$. The profit allocation rule is then computed from expressions (14). The unit resource reward vector, denoted $rr$, takes the value: $rr = [0.50 \quad 1.62 \quad 0.50 \quad 0.90]^T$, and the profit shares of the 4 enterprises of the supply chain are:

$$u_1 = 542.86, \quad u_2 = 150.44, \quad u_3 = 160.00, \quad u_4 = 262.41.$$ 

It is observed that all the partners of the chain obtain a strictly positive profit share.

If the member enterprises were rewarded only on the basis of the shadow prices of their resources, the profit share of the third enterprise would have dropped to zero. This is because only resources 2 and 5 have a strictly positive shadow price and enterprise 3 does not own any quantity of these resources.

In the proposed allocation rule, any resource $r$ is rewarded not only on the basis of its shadow prices, $z^*_r$, but also its utilization cost measured through its WIP marginal cost, $\eta^2_r$.

5. CONCLUSIONS

In this study, we have analyzed the problem of supply chain design through formulation of a cooperative game named PLSCG (Piecewise Linear Supply Chain Game). For this purpose, a supply chain has been modelled as a coalition of partners pooling their resources and sharing the same profit function. The supply chain model has been formulated from the multi-level manufacturing process and the resources used for producing components and end-products. The best use of resources and the best mix of products have been computed by Linear Programming. They generate the maximal expected profit for the supply chain. Then, a profit allocation technique has been computed. It derives from the dual of the profit maximization problem and belongs to the core of the SCG. This result generalizes the properties of the Owen set for the classical LPG (Linear Production Game). However, the use of piecewise linear clearing functions for resources and the introduction of additional variables to represent the workloads have improved the stability of coalition by better rewarding the utilization of resources.

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