Finite elements reduced order models for nonlinear vibrations of stratified piezoelectric beams with application to NEMS

A. Lazarus, O. Thomas, J.-F. Deü

Structural Mechanics and Coupled Systems Laboratory,
Cnam Paris, France
lazarus@ladyhux.polytechnique.fr, {olivier.thomas, jean-francois.deu}@cnam.fr

Figure 1: (Left, middle) Scanning electron microscope picture and sketch of typical clamped/clamped stratified beams (Right) Beam resonance curve, for several excitation levels (’—’:: stable solution; ’- -’:: unstable solution).

The large amplitude non-linear vibratory behavior of a stratified straight beam with piezoelectric patches is addressed in this study. The main goal is to build an efficient model of a NEMS (Nano Electro Mechanical System) resonator, that can be used in various applications, such as mass sensors [1], bit storage devices [2] and ultra-high frequency (UHF) resonators [3]. All those devices are based on several beam resonators equipped with one or more piezoelectric films for sensing the vibrations and/or driving them in vibration.

Two main issues have to be modeled. At first, the presence of the piezoelectric films as well as the manufacturing constraints impose several layers to the structure, each one composed of a given material, so that the beam structure is stratified. An example is shown on Fig. 1. Secondly, geometrically non-linear vibratory phenomena have to be modeled, for two main reasons. The first one is related to clamped/clamped boundary conditions associated to large length-to-thickness ratio, that create curved “Duffing-like” resonance curves for moderate excitation levels, within the using range of the devices [1] (Fig. 1). The second one is related to particular applications where the device is subjected to parametric excitation. In this case, an axial harmonic excitation at twice a resonance frequency creates transverse vibration at the resonance frequency [2].

The main purpose of this work is to build an efficient model that takes into account the above mentioned issues, to serve in the design process of the resonators. In particular, quantifying the amount of non-linear effects on the vibratory characteristics such as the resonance frequencies, predicting possible jump phenomena and also precisely evaluating the parameter ranges that lead to parametric excitations is essential. For those reasons, the model must be precise, predictive but also reduced, to minimize the computational costs and to enable parameter variation studies.
At present, various analytical models have been proposed, based on classical von Kármán assumptions (to include the geometrical non-linearities) and an Euler-Bernoulli kinematics across all layers [4] to take into account the stratification. The obtained partial differential equations are then discretized using a normal mode expansion, thus allowing the use of continuation methods to calculate the resonance curves. On the other hand, modern numerical techniques, based on the finite elements methods for instance, can tackle those problems. However, the dynamics is often obtained by time integration of the non-linear discretized problem, leading to very time consuming simulations to obtain a single resonance curve, since one has to wait for the steady state regime for each excitation frequency.

In this work, we propose a numerical model based on a finite-elements discretization of the beam geometry, using a non-linear dynamic formulation of a total Lagrangian plane beam element [5]. This formulation is interesting since it is valid for large amplitude vibration, with no restrictions on the rotation amplitude. It thus enhances the von Kármán analytical model application range, restricted to moderate rotations. The stratification is included thanks to an Euler-Bernoulli kinematics across all layers, in the same manner than in [6].

The main model size reduction strategy is to use a normal mode expansion of the solution, which is truncated to a small number of modes. To do this within the finite elements framework, three strategies are tested. The first one consists in performing a time integration of the reduced order model (ROM) using a Newmark method combined with a Picard iteration scheme. Even if it is based on a direct time integration like in the full order approach, it is less time consuming since the size of the ROM is much lower than that of the initial finite-element model. The second strategy is based on a non-intrusive technique, introduced in [7]. It consists in calculating all the coefficients of the ROM by evaluating the internal force vector $f_{int}(X)$ of the discretized equation of motion with special choices for the displacement vector $X$. Finally, the third technique is an original way of calculating the ROM coefficients, at an elementary level. For the last two techniques, any continuation method, such as the asymptotic numerical method [8], can be applied to calculate the resonance curves. For all three strategies, the key point that is addressed is the choice of the normal modes retained in the expansion basis.

References


